Homework assignment 6

Determine the motion of the double pendulum from assignment 1 by numerical integration of the equations of motion expressed in independent generalized coordinates as derived in assignment 5. The initial conditions are both bars vertically up at zero speed. We assume a gravitational field operating in the horizontal direction with a field strength of $g = 9.81 \text{ N/kg}$. We want to determine the angle, in radians, of both bars with respect to the horizontal axis after 3.0 seconds with a maximal absolute error of $10^{-6}$ rad.

a. Determine the accordingly maximum step size for the following numerical integration methods:

(1) Euler.
(2) Heun.
(3) Runge-Kutta $3^{rd}$ order.
(4) Classical Runge-Kutta $4^{th}$ order.

Use an error estimate method based on the method-inherent truncation error and the round-off error due to the finite precision as explained in the course. Plot for each angle in one figure the $\log_{10}(\text{estimated error})$ versus the $\log_{10}(\text{step size})$ for all four methods. In some cases it may not be feasible to find such a maximum step size. Either the method used is unstable or the step size becomes so small that it is impractical to reach the end point at $t = 3.0$ seconds. If so, please do not spend too much time on that and state your case clearly!

b. Finally use the three ODE solvers $\texttt{ode23}$, $\texttt{ode45}$, and $\texttt{ode113}$ from Matlab. Set the error tolerance $\texttt{RelTol}$ and $\texttt{AbsTol}$ such that you get the same final accuracy (global error!) as above and integrate the equations of motion for 3.0 seconds.

(1) Compare the angles of both bars at $t = 3.0$ sec with the results from above.
(2) Determine the average step size and the total number of function evaluations (calls to the differential equation $f(t,y)$) as used in the three methods. Do these agree with your previous results?

Please tabulate all your results (with enough digits [for instance 15] to show convergence) and discuss.

**Bonus Question:** What happens when you start a simulation with initial conditions very close to those from above? Do the solutions stay close together? How do we call such a behaviour?