A rigid body on a skate: another device that cannot stand still but balances while moving

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Abstract

Passive mechanical systems that are dynamically stable in statically unstable configurations include tops, a passive bicycle[1], a skateboard with a rigid rider[2], a polygon rolling downhill (in 2D and in 3D), and some simple models of walking[3]. These systems gain their stability by a mixture of having fast-spinning parts, dissipation, having nonholonomic kinematic constraints, having intermittent contact, and having linkages that connect internal degrees of freedom.

The purpose of this work is to find stability of a statically unstable configuration with the fewest number of these features. We study a single three-dimensional rigid body supported by a single-point skate contact on a level plane. Thus there is no dissipation, no fast-spinning parts, no intermittent contact, and no internal linkages and internal degrees of freedom. This system is conservative but non-Hamiltonian because of the nonholonomic skate constraint. The only way to achieve stability is through adjusting the mass distribution.

Using linearized stability analyses and fully nonlinear numerical simulations, we investigate whether such a body can have asymptotically stable motions (in some variables). This system can be thought of as a model of an ice skater on a single ice skate or of a person riding a massless monocycle. Of interest is the possible role of mass distribution for the qualitative dynamics of, say, vehicles, robot, and prosthetic limb designs.

1. R. S. Hand, Masters thesis, Cornell University, Ithaca, NY, 1988.
2. M. Hubbard, J. Appl. Mech. 46, 931 (1979).
3. M. J. Coleman, M. Garcia, K. Mombaur, and A. Ruina. Prediction of stable walking for a toy that cannot stand. Phys. Rev. E, 64(2):22901(3), 2001.