TAM 674
Applied Multibody Dynamics

Spring Term 2003, Mon & Wed 10:10-11:00, 202 Thurston Hall, 3 credits.

Homework assignment 7

Determine the motion of the double pendulum from assignment 1 by numerical integration of the equations of motion as derived in assignment 5 or 6. The initial conditions are both bars vertically up at zero speed. We assume a gravitational field operating in the *horizontal* direction with a field strength of $g = 9.81$ N/kg. We want to determine the angle, in radians, of both bars with respect to the horizontal axis after 5 seconds with a maximal absolute error of 10^{-5} rad. Try and find the accordingly maximum step size for the following numerical integration methods:

1. Euler.
2. Heun.
3. Runge-Kutta 3^{rd} order.
4. Classic Runge-Kutta 4^{th} order.
5. Euler for second order differential equations.

Use the error estimate method as explained in the course and plot the $\log_{10}$(estimated error) versus the $\log_{10}$(step size) for all different applied methods in one figure.

Now use the ODE solvers `ode23`, `ode45`, and `ode113` from Matlab. Set the error tolerance `RelTol` and `AbsTol` accordingly and integrate the equations of motion from $t = 0$ until $t = 5$ seconds. Compare the angle of both bars with the results from above and determine the average step size and the total number of function evaluations (calls to f(t,y)) as used in the three methods. Do these agree with your previous results? Please discuss.

References

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[3] L. F. Shampine and M. K. Gordon. *Computer solution of ordinary differential equations: the initial value problem.* W. H. Freeman, San Francisco, 1975.

[4] J. C. Butcher. *The numerical analysis of ordinary differential equations: Runge-Kutta and general linear methods.* J. Wiley, Chichester, New York, 1987.

[5] J. H. Hubbard and B. H. West. *Differential Equations: A Dynamical System Approach, Part 1, Ordinary Differential Equations.* Number 5 in Texts in Applied Mathematics. Springer, New York, 1991.

[6] E. Hairer, S. P. Nørsett, and G. Wanner. *Solving Ordinary Differential Equations I: Nonstiff Problems.* Number 8 in Springer Series in Computational Mathematics. Springer-Verlag, Berlin Heidelberg, second revised edition, 1993.

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[9] R. von Schwerin. *Multibody System Simulation: Numerical Methods, Algorithms, and Software.* Number 7 in Lecture Notes in Computational Science and Engineering. Springer-Verlag, 1999.