TAM 674
Applied Multibody Dynamics

Spring Term 2003, Mon & Wed 10:10-11:00, 202 Thurston Hall, 3 credits.

Homework assignment 4

Consider the following three models:

a. Add a spring to the model from homework assignment 1. The spring is attached to the ground in D, with coordinates $(-l/2, 0)$. The other side of the spring is connected to bar 1 in point E, being at 2/3 of the length measured from point A. The free length of the spring is $l_0 = 2l/3$ and the linear stiffness is $k = (15/2)(mg/l)$.

   Calculate the the accellerations of the center of mass of the two bodies together with the Lagrangian multipliers for the case where the two bars are vertical up and zero speed.

b. Add a motor to the model from homework assignment 1. The motor is placed in the cylindrical hinge between the ground and bar 1. We assume the motor to have a constant speed of $\omega = 50$ rpm.

   Calculate the the accellerations of the center of mass of the two bodies together with the Lagrangian multipliers for the case where the two bars are vertical up and with an initial angular speed of ω on both bars.

c. Add an impulsive contact to the model from homework assignment 1. We assume that the right end of body 2, point C, hits a vertical wall. This vertical wall is horizontal positioned in the origin. We assume that both bars are vertical up which means that the contact will occur in point D with coordinates $(0, 2l)$, furthermore we assume an angular speed of $\omega = 50$ rpm on both bars. Consider the case were the coefficient of restitution e is respectively 1, 0.9 and 0. Calculate for these different values:

   (1) The velocities after impact of the centre of mass of the two bars together with the contact impulse and the joint constraint impulses.

   (2) The difference in the kinetic energy of the system before and after the impact and the amount of work done by the contact impulse.

The impulsive equations for the constrained multibody system are usually written as a set of linear equations in the unknown velocities of the centre of mass after impact $\dot{x}_i^+$, and the contact impulses and constraint impulses ρ_k. Instead of the velocities $\dot{x}_i^+$ we could use the velocities jumps $\Delta\dot{x}_i = \dot{x}_i^+ - \dot{x}_i^-$ as unkowns.

d. Rewrite the impulsive equations in terms of these unknown velocity jumps and the contact and constraint impulses and discuss the computational pro's and con's.

We want to address the energy loss during impact in a constrained multibody system and therefore we compare the kinetic energy of the system before and after impact in the absence of applied impulses p_i.

e. Show that the energy loss due to the impact is equal to $\frac{1}{2}(1-e)\rho_c\dot{\varepsilon}_c^-$, with the contact impulses ρ_c and the relative velocities $\dot{\varepsilon}_c^-$ before impact at the contact points.