1974

Calspan

A NOTE ON DESIGN CRITERIA FOR BICYCLE STABILITY IN TERMS OF FRONT END GEOMETRY

by:

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December 1974

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In the last several years there has been a tremendous increase of interest in bicycles and with this interest has come a concern about bicycle safety. It is generally agreed that the safety question has many aspects and a great many people are involved in various approaches to its solution. One of these approaches calls for the formulation of bicycle design and performance standards. Specifications regarding bicycle stability are being considered for incorporation as part of these standards. This note is an attempt to compare some of the current concepts of bicycle stability requirements. Before discussing specific approaches, a brief general review of bicycle front end geometry may be useful.

One of the problems in preparing this note was that of nomenclature and identification of terms. Head tube angles are measured with respect to either vertical or horizontal; head tube angles are frequently called rake angles, but rake is sometimes used in place of geometrical offset; offset may be referenced to either geometry or mass; a variant of offset, called front projection, is used; and trail (in the physics of the problem, a most important parameter) is defined in two different ways. Figure 1 attempts to show these terms in their proper relationships as used in this note. It should be pointed out that

1. trail (t) is measured perpendicular to the steer axis. Although it is more common to measure trail along the ground line, its physical significance to stability is with respect to its function in contributing to moments about the steer axis.

2. head tube angle (θ) is measured with respect to a horizontal (ground) line. Its complement (σ) is, however, a more convenient term for use in the expressions which are derived in this note. In all cases, $\theta + \sigma = 90$ (degrees).

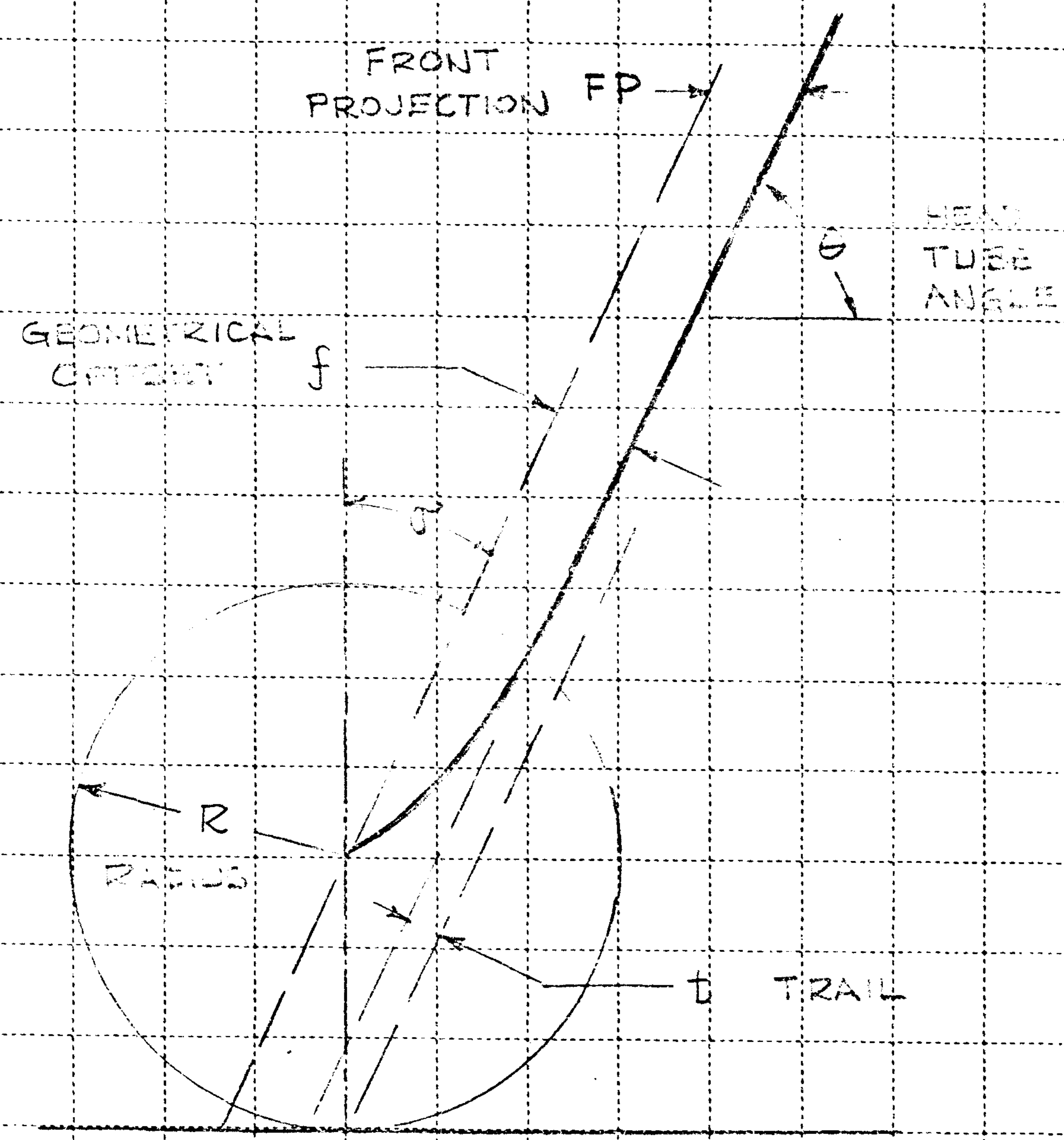

FIGURE 1

BICYCLE FRONT END GEOMETRY

3. geometrical offset (f) is the perpendicular distance between the steer axis and a line parallel to it passing through the wheel center.

4. a dimension which has been called front projection (FP) is simply geometrical offset (f) multiplied by csc θ. In terms of the primary variables used in this note,

$$FP \cos \sigma = f$$

5. the sum of offset (f) and trail (t) as defined here is simply $R \cos \theta$ or $R \sin \sigma$.

Additional design and operational variables are also significant to the overall stability characteristics of a bicycle (and to some of the stability criteria) but they will be defined as required in the discussion.

Four design approaches have been reviewed. They are: (1) the proposed ISO standard; (2) the Davison formula; (3) David Jones' stability parameter; and (4) the Schwinn/Calspan stability index. They are described below.

Proposed ISO Design Standards

The International Standards Organization (ISO), Committee 149, has under consideration the adoption of bicycle design standards calling for limitations on steering assembly geometry (1). This standard would restrict head tube angles to the range of 65-75 degrees and require the projection of the steering axis to intersect a vertical line from the front axle within a range of 15% to 60% of the wheel radius above the ground contact point. Figure 2 depicts this requirement.

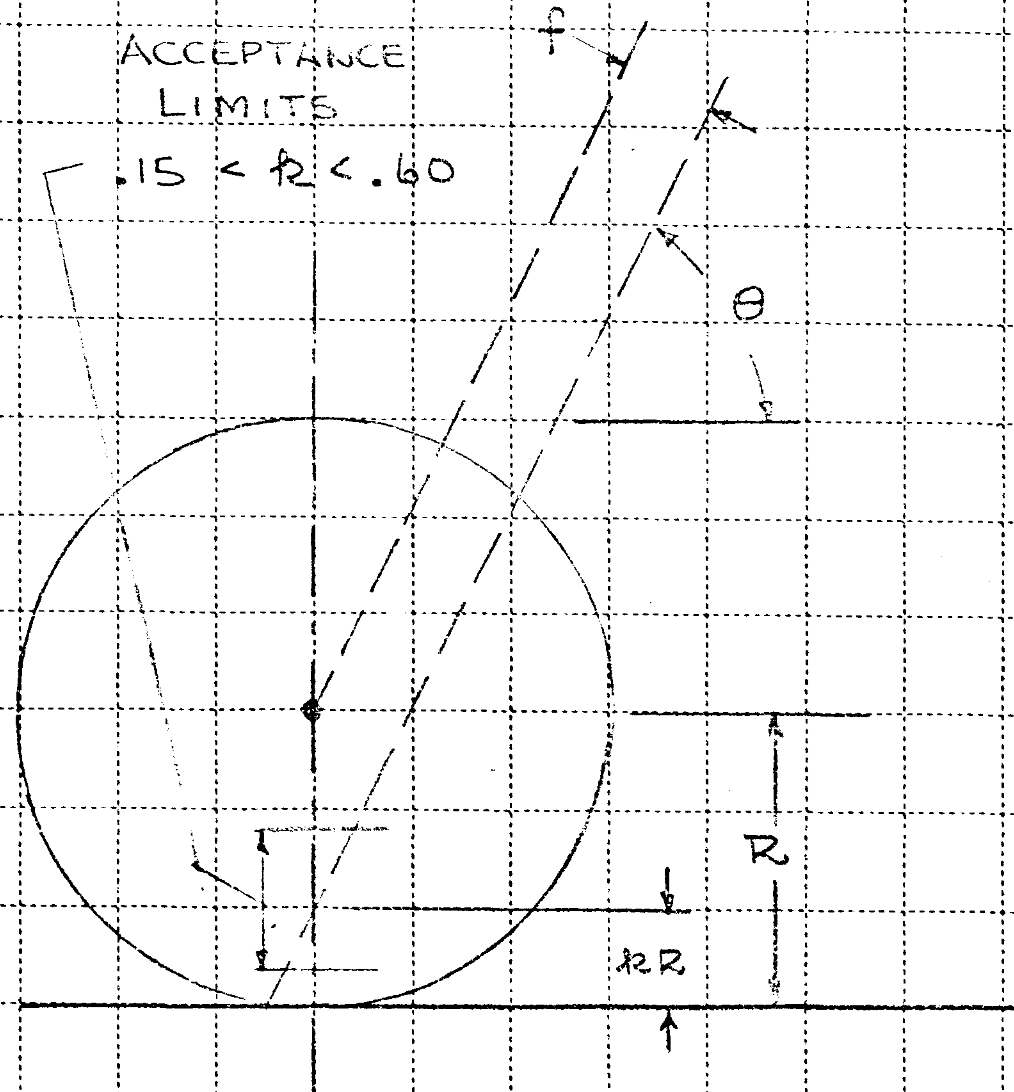

FIGURE 2

ISO DESIGN REQUIREMENTS

Described in terms of geometrical offset of the front fork, this specification can be written as

$$f = R\ (1 - k)\cos\theta \qquad (1)$$

where f is geometrical offset, R is wheel radius, θ is head tube angle, and k is the ratio of the intersection point to the wheel radius. Simplifying by replacing cos θ with its complementary term, sin σ, and then using the small angle approximation (sin A = A) with only about 3% error over the range of interest, equation (1) becomes:

$$f = R\ (1 - k)\ \sigma$$

For the two extremes in the value of k (.15 and .60) and for R = 13.5 inches, the limiting curves for offset are

$$f_{.15} = 11.5\ \sigma \text{ in. } (\sigma \text{ in radians}) = .2\ \sigma \text{ in. } (\sigma \text{ in degrees}) \qquad (2)$$

$$f_{.60} = 5.4\ \sigma \text{ in. } (\sigma \text{ in radians}) = .095\ \sigma \text{ in. } (\sigma \text{ in degrees})$$

Thus, this standard suggests a range of acceptable values for offset which decreases with head tube angle. At any given angle, the ratio of the largest acceptable offset to the smallest is approximately 2. It will be noted that the smaller value (i.e., the $f_{.60}$ value) provides larger trails and therefore greater stability.

Davison's formula

In his book on bicycling (2), Delong cites an expression for front end geometry design which he identifies as the Davison formula. It is

$$f = R\tan\frac{(90 - \theta)}{2} \qquad (3)$$

where f is geometrical offset, R is front wheel radius, and θ is head tube angle. For values of θ around 70 degrees, the small angle approximation of tan A = A (in radians) can be made with an error of less than 2%, and the expression can be simplified and linearized to

$$f = \frac{R\sigma}{2} \text{ (} \sigma \text{ in radians) } = \frac{R\ \sigma}{114.6} \text{ (} \sigma \text{ in degrees)}$$

where $\sigma = 90 - \theta$ (σ in degrees). With a 27 inch wheel, the final form is

$$f = .118\ \sigma \quad (4)$$

where f is in inches and σ in degrees.

The principal observation to be made about this design criterion is that the total displacement term, R sin σ , is about equally divided between offset and trail for all reasonable values of head tube angle. Note also that the value of offset determined by this formula is approximately 25% larger than that given by the smaller of the two ISO expressions, equation (2).

Jones' Stability parameter

As part of his widely-known article on unrideable bicycles (3), David Jones developed a graphic representation of the stability of bicycles based on a parameter involving motion of a point on the fork as a function of steer and lean angles. This parameter, which Jones identifies as $\partial^2 H / \partial \alpha \partial L$*, will not be discussed in detail here. Instead, it will be sufficient to utilize data from Figure 5 of Jones' article to obtain an expression for stability as a function of head tube angle and geometrical offset. (In this figure, which is shown in slightly modified form in this note as Figure 3, Jones actually uses the complement of head tube angle and a term he calls front projection.)

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* Yet another set of symbols!

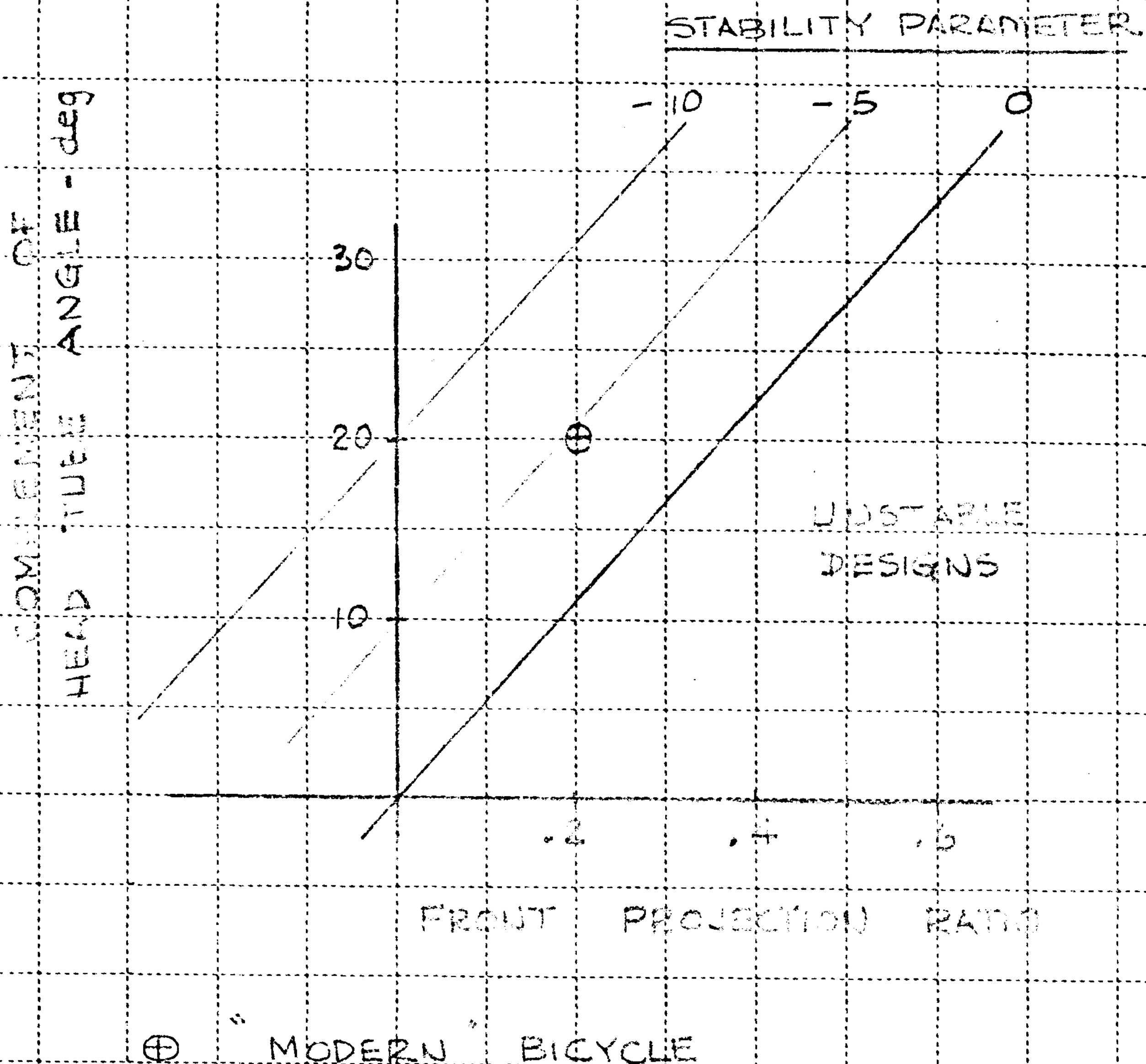

FIGURE 3
JONES' STABILITY PARAMETER

The equation of any line of constant stability value can be determined to be approximately

$$F \approx .036\,(SP) + .0183\,\sigma \qquad (5)$$

where F is front projection (as a percentage of wheel radius); (SP) is the stability parameter value (negative for stable configurations); and σ is the complement of the head tube angle (in degrees). Since

$F = \frac{[illegible]}{R} = \frac{[illegible]}{R}$, equation (5) can be written as

$$f = R \cos\sigma\,(.0183\,\sigma + .036\,(SP))$$

For a 27 inch wheel and with $\cos\sigma$ assigned an average value over the range of interest of .94 (i.e., cos 20°) the simplified expression becomes

$$f = .233\,\sigma + .46\,(SP) \qquad (6)$$

The first term on the right hand side of the equation is the expression for "zero stability". Its value is about 16-17% larger than the maximum acceptable value for offset according to the ISO standard, equation (2). The second term, which specifies how much stability is designed into the bicycle and may be related to trail, merely reduces the value of offset as stability is increased.

Schwinn/Calspan Stability Index

One of the principal outputs of a simplified analysis of bicycle dynamics performed recently by Calspan for Schwinn (4) was the identification of a specific value of velocity for which pure steering torque control is impossible. This velocity, which was called the inversion speed, is determined by several design and operational variables of the bicycle-rider system. In a somewhat simplified form, it may be expressed as

$$V_I^2 = \frac{(Z_F t - W_s f_m) \ell R}{i_F \cos \sigma} \qquad (7)$$

where:

V_I = inversion speed (a stability index)

Z_F = vertical force (load) on the front wheel

t = trail

W_S = weight of steering assembly

f_m = mass offset of steering assembly

ℓ = wheelbase

R = front wheel radius

i_F = moment of inertia of front wheel about its spin axis

σ = complement of head tube angle

If it is assumed that $Z_F t >> W_S f_m$ and that i_F can be approximated as $\frac{W_S}{g} R^2$, equation (7) can be recast as

$$V_I^2 \; R \cos \sigma = Z_F \ell t \qquad (8)$$

This expression, incidently, points out the greater significance of trail compared to geometrical offset on stability. It further indicates that for a given level of stability in a specific bicycle (i.e., a selected value of V_I), the value of trail changes only in proportion to cos σ . Since the value of cos σ changes very little over the range of head tube angles of interest (say, from 80 to 60 degrees), the recommended value of trail is relatively constant and independent of head tube angle. Thus, most of the change in steering layout occurs in the value of geometrical offset as head tube angle is changed.

To convert equation (8) to one involving offset, use is made of the fact that $f + t = R \sin \sigma$ and the small angle approximation is applied. Then,

$$t = R\sigma - f = \frac{V_I^2 R \cos\sigma}{Z_R \ell}$$

With a 27 inch wheel,

$$f = .235 \ \sigma - KV_I^2 \qquad (9)$$

where K is a constant involving the remaining design and stability terms which, as mentioned earlier, has been somewhat simplified for use here.

Equation (9) has the same form as the expression derived from Jones' work, equation (6). The first term on the right side would have exactly the same value as that in equation (6) except for arithmetic round-off errors. When $f = .235\ \sigma$, the value of trail is zero and the value of the inversion speed must therefore also be zero. Increased stability is reflected as increased inversion speed, increased trail, and reduced offset - as shown by the equation. The Jones stability parameter and the simplified Schwinn/Calspan index appear very similar, but the complete Schwinn/Calspan index is more comprehensive because of its inclusion of additional design and operational factors.

Figure 4 shows these various criteria plotted with respect to coordinates of geometrical offset and head tube angle. The principal observations to be made from the figure are

1. In the limit (i.e., zero values of designed-in stability), both the Jones criterion and the Schwinn/Calspan index give the same line on this plot. It is the condition for which the value of trail is zero.

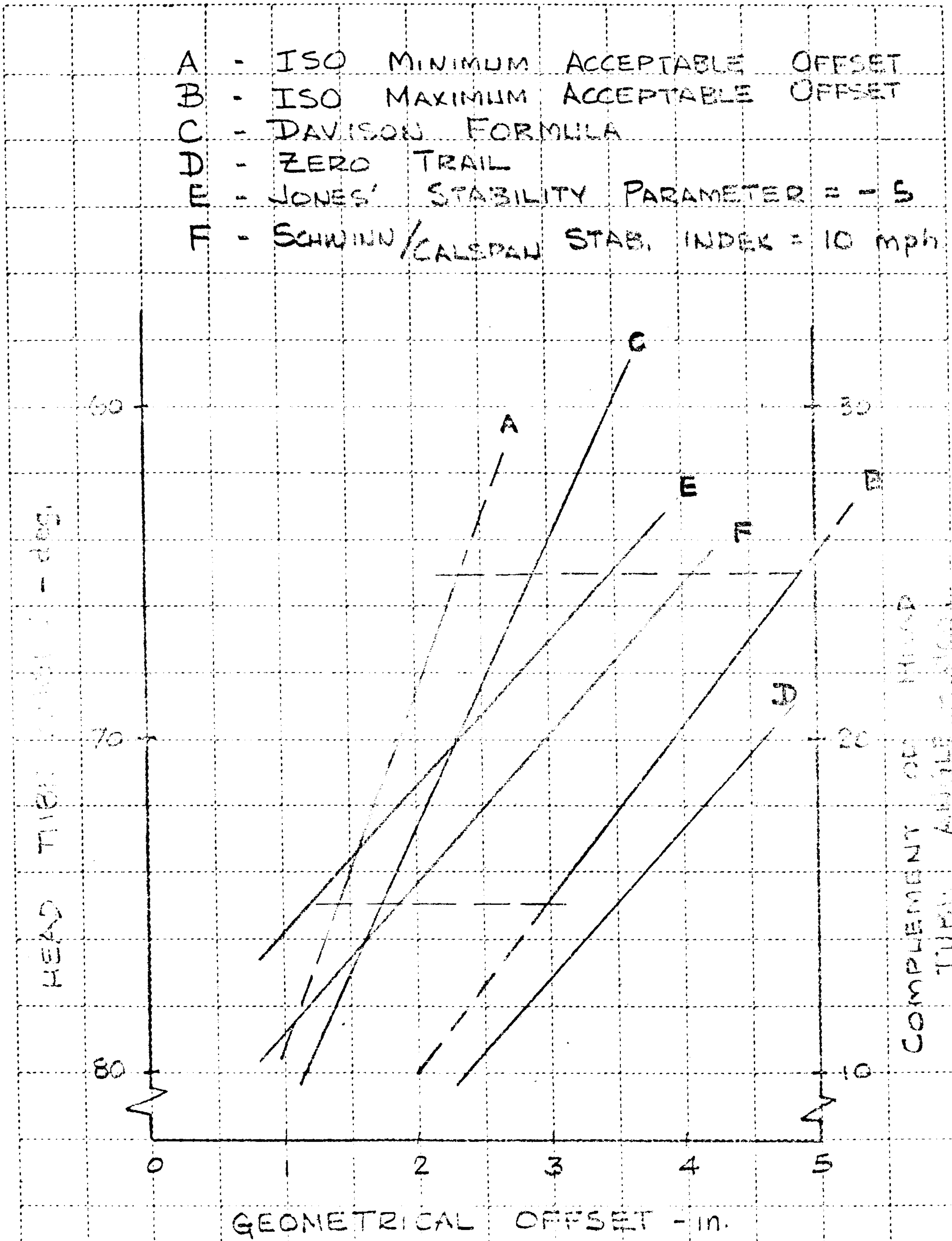

FIGURE 4

STABILITY CRITERIA COMPARISON

2. The lines associated with the minimum acceptable ISO standard and with the Davison formula are quite close together. These two criteria tend to describe very stable bicycles.

3. The lines which represent a reasonable lower limit of stability according to the Schwinn/Calspan index (equivalent to an inversion speed of approximately 10 mph) and a level of stability based on the Jones parameter about the same as for a "modern bicycle" (i.e., reflecting good design practice) lie generally between the ISO limits but have a somewhat different slope than the ISO curves. These are lines F and E, respectively, on Figure 4. Line E is approximately equivalent to a line for an inversion speed of 12 mph using the Schwinn/Calspan index.

4. Values of the design parameters for current premium bicycles tend to lie in the lower left portion of the area (i.e., near line A and between 70 and 75 degrees) bounded by the ISO requirements as shown in the figure. It will be noted that the other criteria (lines C, E, and F) also emphasize this portion of the figure. Thus, designs near line B (the maximum acceptable offset acceptable by the ISO requirements) would seem to provide insufficient stability according to current practice.

Concluding Remarks

This note describes several criteria for the design of bicycle front end geometries with particular emphasis on the contribution of the steering assembly to bicycle stability. It will be recognized that only a part of the overall stability problem is addressed and that additional criteria are needed to define complete design guidelines. Nevertheless, some interesting comparisons between the various criteria and their interpretations are possible.

Within the limitations imposed by the various simplifying assumptions used with the different approaches, first order comparisons may be made in terms of only head tube angle and geometrical offset of the steering assembly. For the purposes of this note, computations have been restricted to the 27 inch wheel but the generalized criteria may be used for any size of wheel. Under some conditions, there is marked similarity among the criteria but the two which relate directly to stability (Jones and Schwinn/Calspan) tend to emphasize the value of trail as the primary design parameters in achieving a given degree of stability.

Equations (2), (4), (6), and (9) are expressions which relate head tube angle and geometric offset for bicycle front end designs. It should be emphasized that all have been somewhat linearized and simplified to facilitate comparison but the errors thereby introduced are believed to be small. Note that equations (6) and (9) contain terms explicitly related to stability so that it should be possible to utilize them for designing bicycles with specific stability characteristics. It remains to be determined, however, what values for these indices are most appropriate.

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3. Jones, David E. H., The Stability of the Bicycle, Physics Today, April 1970.

4. Rice, Roy S., Bicycle Dynamics - Simplified Steady State Response Characteristics and Stability Indices, Calspan Report No. ZN-5431-V-1, June 1974.