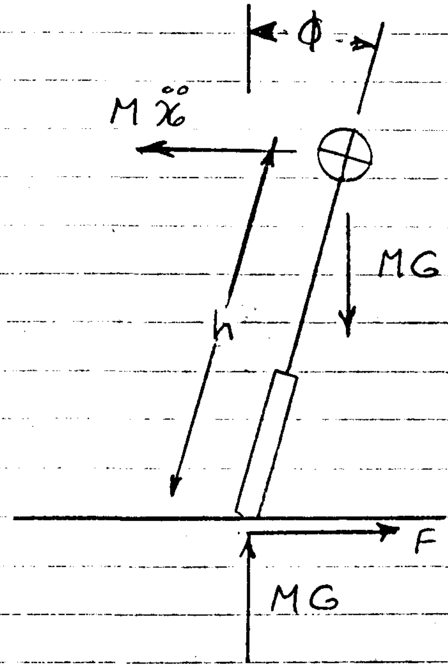
# TIRE FACTORS INFLUENCING THE HANDLING OF CAMBER STABILIZED VEHICLES

CAMBER STABILIZED VEHICLES (UNICYCLE, BICYCLE, ARTICULATED TRICYCLE) OPERATE ON A PRINCIPLE OF BALANCE TO ACHIEVE OVERTURNING STABILITY DURING CORNERING. THE SITUATION CAN BE ILLUSTRATED WITH A UNICYCLE DIAGRAM:




FOR THIS STEADY STATE CONDITION:

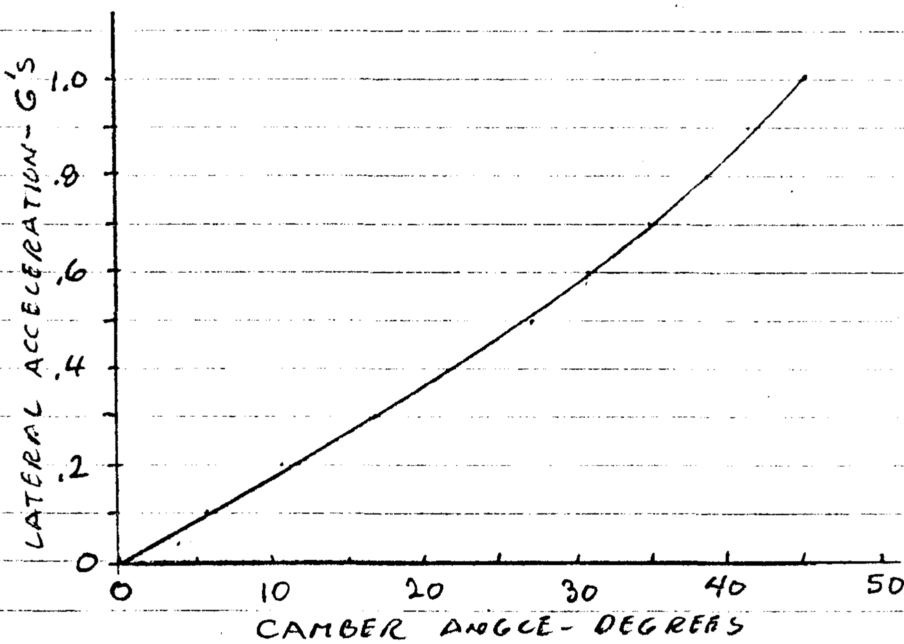
$$M\ddot{x}(h \cos\phi) = MG(h \sin\phi)$$

SO THAT:

$$\frac{\ddot{x}}{G} = \frac{\sin\phi}{\cos\phi} = \tan\phi = a_y$$

THIS RELATIONSHIP TIES THE CAMBER ANGLE TO THE LEVEL OF MANEUVERING SEVERITY SO LONG AS THE RIDER DOES NOT SHIFT HIS WEIGHT AWAY FROM THE VEHICLE'S PLANE OF SYMMETRY.

A GRAPH OF THE CAMBER ANGLE - LATERAL ACCELERATION RELATIONSHIP IS THE FOLLOWING:




A LATERAL FORCE MUST BE DEVELOPED AT THE TIRE-ROAD INTERFACE IN ORDER TO ACHIEVE A LATERAL MANEUVER. THE DEVELOPMENT OF THIS FORCE IS THOUGHT TO BE ASSOCIATED WITH THREE MECHANISMS:

1. DEFLECTION (OR NORMAL FORCE)
2. SLIP ANGLE
3. CAMBER ANGLE

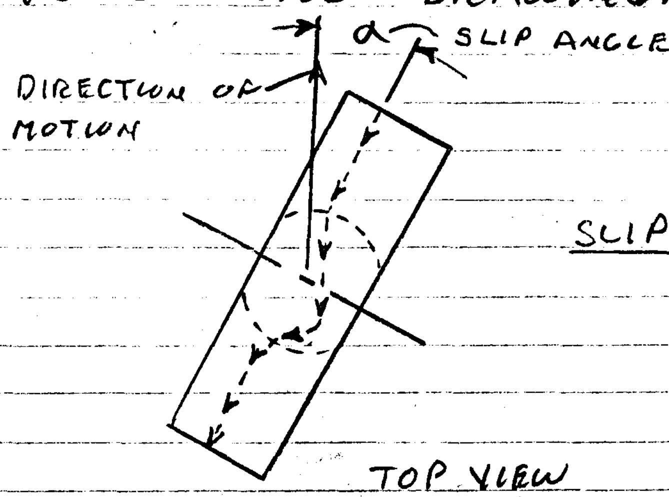
NORMAL FORCE IS IMPORTANT FOR TWO REASONS:

1. IT ALLOWS THE TIRE TO OPERATE AS AN ELASTIC MEMBER WITHIN THE LIMITS IMPOSED BY TIRE-ROAD FRICTION. THIS ELASTIC REGION OF TIRE OPERATION ALLOWS THE DEVELOPMENT OF LATERAL FORCES THAT ARE ROUGHLY PROPORTIONAL TO SLIP ANGLE AND CAMBER ANGLE. THIS PROPORTIONALITY RELATIONSHIP IS NECESSARY FOR VEHICLE

CLOSED LOOP CONTROL. AS LATERAL FORCE AT THE LIMIT OF FRICTION IS APPROACHED, THIS PROPORTIONALITY IS LOST AND THE RIDER LOSES CONTROL.

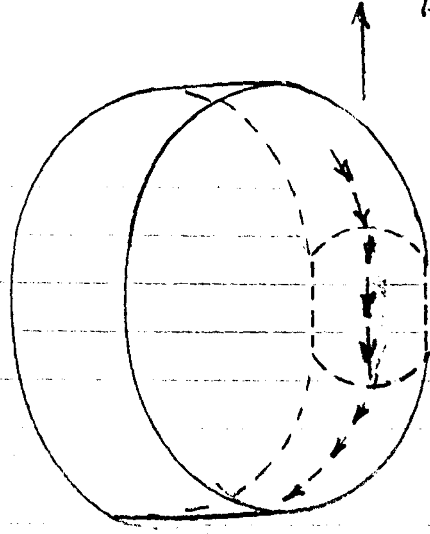
2. DEFLECTION ASSOCIATED WITH NORMAL FORCE PRODUCES A TIRE-ROAD CONTACT AREA WHERE LATERAL FORCE IS DEVELOPED. THE POSITION OF THIS CONTACT AREA, DISTRIBUTION OF PRESSURES, AND ELASTIC DEFORMATIONS ARE THE MECHANISMS CONTROLLING TIRE HANDLING PERFORMANCE.

THE SLIP ANGLE MECHANISM IS ILLUSTRATED BY THE FOLLOWING DRAWING.




SLIP ANGLE MECHANISM

AS EACH ELEMENT OF TREAD ENTERS THE CONTACT REGION IT ADHERES TO THE ROAD AND CONTRIBUTES TO ELASTIC DEFORMATION OF THE TIRE STRUCTURE WHICH PRODUCES A LATERAL FORCE. THE MECHANISM FOR GENERATION OF LATERAL FORCE DUE TO CAMBER ANGLE IS LESS OBVIOUS BUT MAY BE SOMETHING LIKE THAT SHOWN IN THE FOLLOWING:




## CAMBER ANGLE MECHANISM

THE COMBINATION OF NORMAL LOAD AND CAMBER ANGLE CAN BE EXPECTED TO PRODUCE SOME CONTACT DISTORTION AND LATERAL ELASTIC STRAIN IN THE TIRE STRUCTURE, THE AMOUNT OF DISTORTION WOULD APPEAR TO BE LESS PER DEGREE OF CAMBER ANGLE THAN IT IS PER DEGREE OF SLIP ANGLE.

THE RELATIVE INFLUENCE OF CAMBER ANGLE AND SLIP ANGLE CAN BE OBSERVED PHYSICALLY BY RIDING A BIKE WITH ONE FOOT SO THAT IT CAN BE CAMBERED AND STEERED DURING OPERATION IN A MORE OR LESS STRAIGHT PATH. THIS MODE OF OPERATION IS ANALOGOUS TO A TIRE FORCE AND MOMENT TEST WHERE VERTICAL LOADS ARE APPLIED AND LATERAL FORCES MEASURED FOR SLIP ANGLE OR CAMBER ANGLE CONDITIONS. PATH DEVIATIONS FOR VARYING DEGREES OF CAMBER ARE SMALL WHILE STEERING INPUTS THAT PRODUCE SLIP ANGLES RESULT IN SUBSTANTIAL PATH CURVATURE.

TIRE FORCE AND MOMENT DATA TEND TO CONFIRM THIS SUBJECTIVE OBSERVATION. FOR A 6.55-9 TIRE USED EARLIER ON ELECTRIC VEHICLES, LATERAL FORCE FOR ONE DEGREE OF SLIP ANGLE IS ABOUT 97 LB AT A 500 LB. NORMAL FORCE. LATERAL FORCE DUE TO ONE DEGREE OF CAMBER IS 15.5 LB AT THE SAME LOAD OR 16% OF THE SLIP ANGLE VALUE. SIMILAR DATA ARE TABULATED BELOW FOR BICYCLE AND MOTORCYCLE TIRE DATA PROVIDED BY CALSPAN.

| BIKE TIRE (100 LB LOAD) | LATERAL FORCE @ 1° SLIP ANGLE | LATERAL FORCE @ 1° CAMBER ANGLE | % |
|---|---|---|---|
| 24-1¼ BREEZE SPORTS | 17.5 LB. | 0.15 LB | 0.9 |
| 24-1⅜ STRAIGHT SIDE | 18.0 | 0.22 | 1.2 |
| 26-1⅜ STRAIGHT SIDE | 21.0 | 0.12 | 0.6 |
| 2[illegible]-1¼ PUFF ROAD RACER | 17.5 | 0.16 | 0.9 |
| 27-1¼ HP SPORTS TOUR. | 19.0 | 0.15 | 0.8 |
| 27-1¼ BREEZE SPORTS | 20.0 | 0.42 | 2.1 |
| 27-1¼ PUFF ROADRACER N. | 21.0 | 0.36 | 1.7 |
| 27-SEW-UP CLEMENT | 18.0 | 0.14 | 0.8 |
| 27-1¼ PUFF RADIAL | 12.0 | 0.60 | 5.0 |
| 27-1¼ LETOUR | 19.0 | 0.78 | 4.1 |
| | AVG. 18.3 | 0.31 | 1.8 |
| (SMALL CAR TIRES 500 LB LOAD) | | | |
| 6.55-9 GOODYEAR | 97.0 | 15.5 | 16% |
| 6.00-12 FIRESTONE | 98.0 | 15.5 | 16% |
| 5.60-12 FIRESTONE | 85.0 | 13.2 | 15.5 |
| | AVG. 93.3 | 14.7 | 15.8 |
| (MOTORCYCLE TIRES 500 LB. LOAD) | | | |
| 5.[illegible]-16 SUPER EAGLE | 68.0 | 9.8 | 14.4% |
| 5.00-16 SLICK | 74.0 | 11.4 | 15.4% |
| | AVG. 71.0 | 10.6 | 14.9 |

IT IS INTERESTING TO NOTE THAT THE MOTORCYCLE TIRES TESTED BY CALSPAN ARE MORE LIKE CAR TIRES IN THEIR CAMBER-SLIP ANGLE RELATIONSHIP THAN THEY ARE LIKE BICYCLE TIRES. THE MOTORCYCLE TIRE DATA WAS RUN AT 12 PSI AND 18 PSI AS COMPARED TO 20 PSI AND 28 PSI FOR THE CAR TIRES WHICH MAY ACCOUNT FOR THE LOWER CORNERING STIFFNESS. CAMBER STIFFNESSES OF THE BIKE TIRES ARE PROPORTIONALLY LOWER AND QUITE VARIABLE.

TO EXAMINE THIS FURTHER, CONSIDER THE UNICYCLE EXAMPLE CORNERING AT 0.2G. ASSUME A 100 LB. LOAD FOR BIKE TIRES AND A 500 LB LOAD FOR MOTORCYCLE AND SMALL CAR TIRES. CAMBER ANGLE MUST BE APPROXIMATELY 11° ACCORDING TO THE RELATIONSHIP DEVELOPED EARLIER. LATERAL FORCE REQUIRED IS 20 LB FOR THE BIKE TIRES AND 100 LB FOR THE CAR AND MOTORCYCLE TIRES. SINCE THE CAMBER ANGLE IS FIXED BY OVERTURNING STABILITY CONSIDERATIONS, IT IS POSSIBLE TO DETERMINE SLIP ANGLE AND CAMBER ANGLE CONTRIBUTIONS TO LATERAL FORCE DIRECTLY FROM TIRE DATA. SLIP ANGLE REQUIRED FOR 0.2G MANEUVERING CAN ALSO BE DETERMINED.

FORCE CONTRIBUTION FOR 0.2G MANEUVER

| BIKE TIRES (100 LB. LOAD) | FORCE DUE TO SLIP ANGLE | FORCE DUE TO CAMBER ANGLE | SLIP ANGLE | % FORCE DUE TO SLIP ANGLE |
|---|---|---|---|---|
| 24-1 1/4 BREEZE SPORTS | 18.3 | 1.7 | 1.1 | 91 |
| 24-1 3/8 STRAIGHT SIDE | 17.6 | 2.4 | 1.0 | 88 |
| 26-1 3/8 STRAIGHT SIDE | 18.7 | 1.3 | 0.9 | 94 |
| 27-1 1/4 PUFF ROAD RACER | 18.2 | 1.8 | 1.0 | 91 |
| 27-1 1/4 HP SPORTS TOUR | 18.3 | 1.7 | 1.0 | 92 |
| 27-1 1/4 BREEZE SPORTS | 15.4 | 4.6 | 0.7 | 77 |
| 27-1 1/4 PUFF ROADRAC. N. | 16.0 | 4.0 | 0.7 | 80 |
| 27 SEWUP CLEMENT | 18.5 | 1.5 | 1.0 | 93 |
| 27-1 1/4 PUFF RADIAL | 13.4 | 6.6 | 1.1 | 67 |
| 27-1 1/4 LETOUR | 11.4 | 8.6 | 0.6 | 57 |
| (SMALL CAR TIRES 500 LB. LOAD) | | | | |
| 5.35-9 GOODYEAR | — | DATA LIMITED TO 6° | — | — |
| 6.00-12 FIRESTONE | -25 | 125 | -.25° | — |
| 5.60-12 FIRESTONE | -11 | 111 | -.12° | — |
| (MOTORCYCLE TIRES 500 LB. LOAD) | | | | |
| 5.00-16 SUPER EAGLE | 0 | 100 | 0 | — |
| 5.00-16 SLICK | -20 | 120 | -.3° | — |

THE DATA TABULATED ON PAGE 7 SUPPORTS THE CONCLUSIONS STATED IN THE MILLIKEN MEMO OF JANUARY 16. THESE ARE THE FOLLOWING:

1. THE DOMINANT SOURCE OF CORNERING FORCE IN BICYCLE TIRES IS SLIP ANGLE RATHER THAN CAMBER ANGLE.

2. FORCE DUE TO CAMBER ANGLE DOMINATES IN THE CASE OF SMALL PASSENGER CAR TIRES AND TWO MOTORCYCLE TIRES. THIS IS DUE TO THE HIGH CAMBER GAIN OF CAMBER STABILIZED VEHICLES (APPROXIMATELY 50°/G) AND THE PROPORTIONALLY HIGHER CAMBER STIFFNESS OF THESE TIRES.

3. THE DIFFERENCES IN CAMBER STIFFNESSES ARE NOT EASILY EXPLAINED. THE PASSENGER CAR TIRES HAVE ASPECT RATIOS IN THE REGION OF 0.8 AND RELATIVELY FLAT TREAD PROFILES WHILE THE SUPER EAGLE MOTORCYCLE TIRE HAS A ROUND PROFILE SIMILAR TO BIKE PRACTICE. THE MOTORCYCLE SLICK HAS A SOMEWHAT FLATTENED PROFILE. THE MOTORCYCLE TIRES WERE RUN AT LOW INFLATION PRESSURES SO THAT THE PROFILE IN THE CONTACT REGION IS SUBSTANTIALLY FLATTENED. ONE COULD ARGUE FROM THIS AND CONTACT GEOMETRY CONSIDERATIONS THAT A FLATTER TREAD PROFILE TENDS TO PRODUCE HIGHER CAMBER FORCES. WITHOUT FURTHER SUBJECTIVE EVALUATION OF HIGH AND LOW CAMBER STIFFNESS TIRES IT IS DIFFICULT TO ESTABLISH A PREFERENCE.

(THE ABOVE CONCLUSIONS ARE LIMITED TO THE 0-0.2G REGION OF OPERATION. CAR TIRE DATA APPLY TO CAMBER ANGLES LESS THAN 10°)

2-27-75

# A BASIC STEERING SENSITIVITY RELATIONSHIP FOR A CAMBER STABILIZED ARTICULATED TRICYCLE

A LATERAL ACCELERATION - STEER ANGLE RELATIONSHIP IS FUNDAMENTAL TO VEHICLE HANDLING. THE DEVELOPMENT OF THIS RELATIONSHIP NORMALLY BEGINS WITH BASIC VEHICLE TURNING GEOMETRY AS IN THE FIGURE ON PAGE 2 TAKEN FROM A-2730 (A PRIMER ON VEHICLE DIRECTIONAL CONTROL). FROM THESE GEOMETRY CONSIDERATIONS, THE FOLLOWING RELATIONSHIP CAN BE DEVELOPED:
(NOTE: THE TRUSTING READER CAN GO DIRECTLY TO EQUATION (19) ON PAGE 5 AND SKIP THE ALGEBRA.)

$$\delta = 57.3\frac{l}{R} + \alpha_f - \alpha_r \qquad (1)$$

WHERE:

$\delta$ = FRONT STEER ANGLE - DEGREES
$l$ = WHEELBASE - FT.
$R$ = RADIUS OF TURN - FT.
$\alpha_f$ = FRONT SLIP ANGLE - DEGREES
$\alpha_r$ = REAR SLIP ANGLE - DEGREES

THE TOTAL FRONT ($F_f$) AND REAR ($F_r$) LATERAL FORCES REQUIRED FOR TURNING ARE THE FOLLOWING:

$$F_f = \frac{Mb}{l}\frac{u^2}{R} = W_f\frac{u^2}{GR} \qquad (2)$$

$$F_r = \frac{Ma}{l}\frac{u^2}{R} = W_r\frac{u^2}{GR} \qquad (3)$$

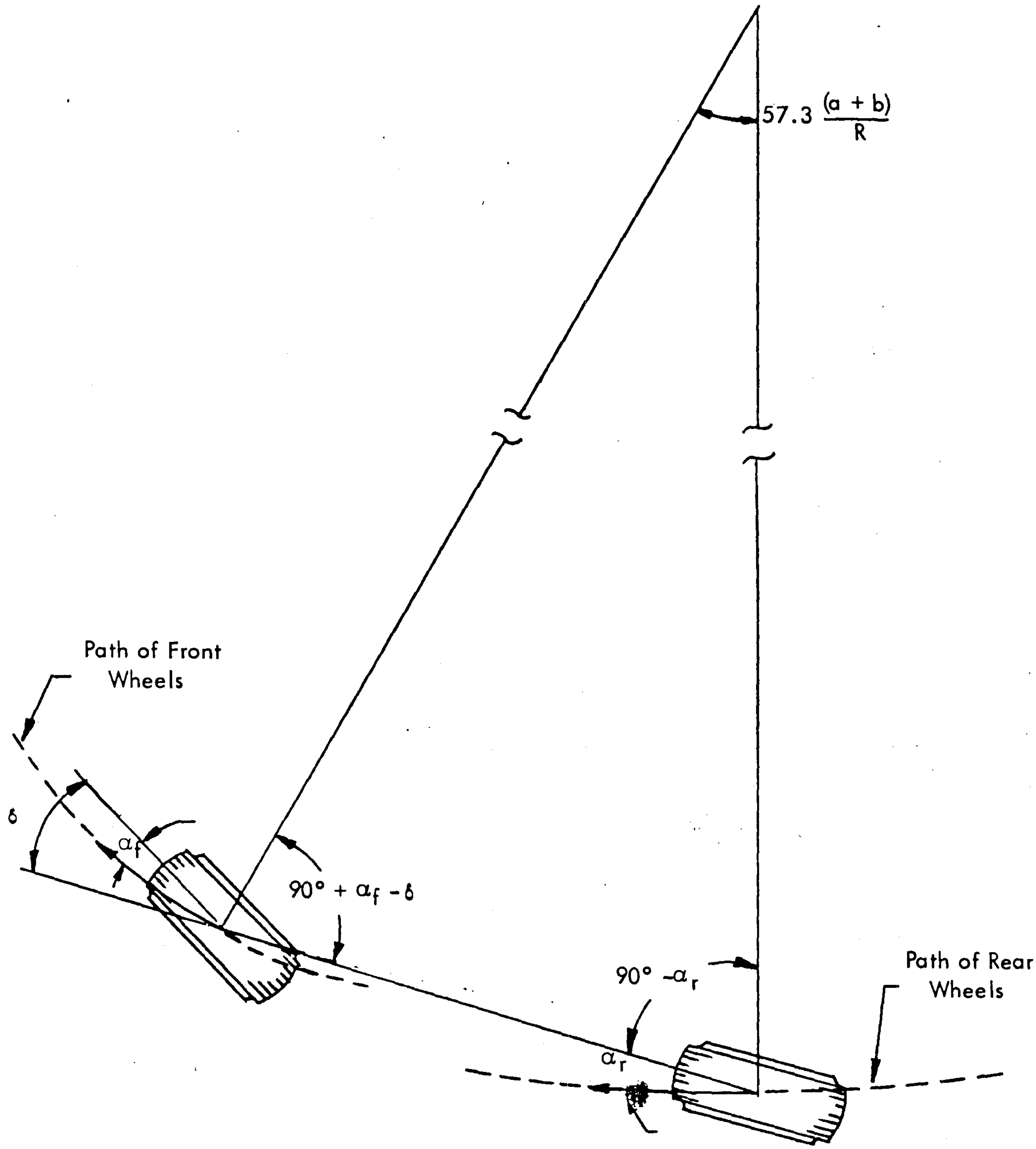



Figure 1 Steer Angle for High Speed Turning

WHERE:

M = VEHICLE MASS - SLUGS
a, b = CENTER OF GRAVITY DIMENSIONS - ft
$\mu$ = SPEED - FT/SEC
G = ACCELERATION DUE TO GRAVITY - ft/sec$^2$
$W_f, W_r$ = FRONT AND REAR TOTAL WEIGHTS - LBS

FOR THIS SUBJECT VEHICLE, TIRE LATERAL FORCES ORIGINATE FROM BOTH CAMBER AND SLIP ANGLE PHENOMENA SO THAT:

$$F_f = F_{\alpha f} + F_{\gamma f} \tag{4}$$

$$F_r = F_{\alpha r} + F_{\gamma r} \tag{5}$$

TO IDENTIFY SEPARATELY CAMBER AND SLIP ANGLE CONTRIBUTIONS. THESE ARE DEFINED FURTHER AS:

$$F_{\alpha f} = C_{\alpha f}\,\alpha_f \tag{6}$$

$$F_{\alpha r} = 2C_{\alpha r}\,\alpha_r \tag{7}$$

$$F_{\gamma f} = \phi\, C_{\gamma f} \tag{8}$$

$$F_{\gamma r} = 2\phi\, C_{\gamma r} \tag{9}$$

WHERE THE C'S ARE SLIP ANGLE AND CAMBER ANGLE STIFFNESSES IN LB/DEG AND $\phi$ IS CAMBER ANGLE. THIS APPROACH, OF COURSE, ASSUMES LINEAR TIRE PROPERTIES AND NEGLECTS FRONT AND REAR CAMBER DIFFERENCES DUE TO FORK GEOMETRY.

SUBSTITUTING EQUATIONS (6), (7), (8), AND (9) INTO (4) AND (5) GIVES:

$$F_f = C_{\alpha f}\,\alpha_f + \phi\, C_{\gamma f} \qquad (10)$$

$$F_r = 2C_{\alpha r}\,\alpha_r + 2\phi\, C_{\gamma r} \qquad (11)$$

COMBINING (2) WITH (10) AND (3) WITH (11) YIELDS:

$$W_f \frac{u^2}{GR} = C_{\alpha f}\,\alpha_f + \phi\, C_{\gamma f} \qquad (12)$$

$$W_r \frac{u^2}{GR} = 2C_{\alpha r}\,\alpha_r + 2\phi\, C_{\gamma r} \qquad (13)$$

SOLVING FOR SLIP ANGLE:

$$\alpha_f = \left(\frac{W_f}{C_{\alpha f}}\right)\left(\frac{u^2}{GR}\right) - \phi\,\frac{C_{\gamma f}}{C_{\alpha f}} \qquad (14)$$

$$\alpha_r = \left(\frac{W_r}{2C_{\alpha r}}\right)\left(\frac{u^2}{GR}\right) - \phi\,\frac{C_{\gamma r}}{C_{\alpha r}} \qquad (15)$$

SUBSTITUTING INTO (1) GIVES THE FOLLOWING:

$$\delta = 57.3\,\frac{l}{R} + \left(\frac{W_f}{C_{\alpha f}}\right)\left(\frac{u^2}{GR}\right) - \phi\,\frac{C_{\gamma f}}{C_{\alpha f}} - \left(\frac{W_r}{2C_{\alpha r}}\right)\left(\frac{u^2}{GR}\right) + \phi\,\frac{C_{\gamma r}}{C_{\alpha r}} \qquad (16)$$

CLEANING UP:

$$\delta = 57.3\,\frac{l}{R} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right]\frac{u^2}{GR} + \left[\frac{C_{\gamma r}}{C_{\alpha r}} - \frac{C_{\gamma f}}{C_{\alpha f}}\right]\phi \qquad (17)$$

THE RELATIONSHIP BETWEEN LATERAL ACCELERATION IN G'S $\left(\frac{u^2}{GR}\right)$ AND CAMBER ANGLE WAS DEVELOPED EARLIER SO THAT:

$$\phi = \mathrm{TAN}^{-1} \frac{\mu^2}{GR} \tag{18}$$

SO FINALLY:

$$\delta = \underbrace{57.3 \frac{l}{R}}_{\substack{\text{ACKERMANN} \\ \text{GEOMETRY}}} + \underbrace{\left[ \frac{W_f}{C_{\alpha f}} - \frac{W_r}{2 C_{\alpha r}} \right]}_{\substack{\text{WEIGHT AND} \\ \text{TIRE SLIP} \\ \text{ANGLE} \\ \text{EFFECT}}} \underbrace{\frac{\mu^2}{GR}}_{\substack{\text{G'S} \\ \text{LATERAL} \\ \text{ACCELERATION}}} + \underbrace{\left[ \frac{C_{\gamma r}}{C_{\alpha r}} - \frac{C_{\gamma f}}{C_{\alpha f}} \right]}_{\text{CAMBER EFFECT}} \underbrace{\mathrm{TAN}^{-1} \frac{\mu^2}{GR}}_{\text{CAMBER ANGLE}} \tag{19}$$

THIS RATHER REMARKABLE RELATIONSHIP IMPLIES THAT TIRE CAMBER PROPERTIES MAY NOT BE INVOLVED IN VEHICLE STEERING SENSITIVITY SO LONG AS THE RATIOS OF CAMBER STIFFNESS TO CORNERING STIFFNESS ARE THE SAME IN THE FRONT AND THE REAR. THIS IS, OF COURSE, SUBJECT TO MANY SIMPLIFYING ASSUMPTIONS AND IS ONE OF SEVERAL POTENTIAL INDICATORS OF HANDLING PERFORMANCE.

ARE SUMMARIZED ON PAGE 13.

LET US NOW COMPARE SOME GAIN VALUES FOR VARIOUS CLASSES OF VEHICLE.

CASE #1 - BICYCLE, 24 - 1 3/8 STRAIGHT SIDE TIRE. MODIFY EQUATION (19) FOR ONE REAR WHEEL.

ASSUME 30 LB BIKE, 200 LB RIDER, WEIGHT DISTRIBUTION 40% FRONT, 60% REAR

$W_f = .4 \times 230 = 92$
$W_r = .6 \times 230 = 138$
$\ell = 4$ FT
$R = 30$ FT
$u = 5$ MPH, 10 MPH = 7.3 FT/SEC, 14.7 FT/SEC
$C_{\alpha f} = 21$ LB/DEG
$C_{\alpha r} = 21$ LB/DEG
$C_{\gamma f} = .12$ LB/DEG
$C_{\gamma r} = .12$ LB/DEG

$$\frac{u^2}{GR} = .055, .219 \text{ g's}$$

$$\delta_{5MPH} = 57.3\left(\frac{4}{30}\right) + \left[\frac{92}{21} - \frac{138}{21}\right](.055) + \left[\frac{.12}{21} - \frac{.12}{21}\right] 3^\circ$$

$$= 7.64 + \left[4.38 - 6.57\right](.055)$$

$$= 7.64 + (-2.19)(.055) = 7.64 - .12 = 7.52^\circ$$ STEER ANGLE AT .055 G's

$$\delta_{10MPH} = 7.64 + (-2.19)(.219) = 7.64 - .48 = 7.16^\circ$$ STEER ANGLE AT .219 G's

BIKE MAY THEREFORE HAVE 2% OF OVER STEER IN THE 0 - .2 G RANGE BUT ACKERMANN EFFECTS TEND TO WASH THIS OUT.

CASE # 2 - MOTORCYCLE, 5.00-16 SUPER EAGLE TIRE, WEIGHT DISTRIBUTION 40% FRONT, 60% REAR. (SINGLE REAR WHEEL)

ASSUME 1000 LB VEHICLE AND RIDER.

$W_f = .4 \times 1000 = 400$ LB.
$W_r = .6 \times 1000 = 600$ LB.
$\ell = 5$ FT.
$R = 30$ FT.
$u = 5$ MPH, 10 MPH $= 7.3$ FT/SEC, 14.7 FT/SEC
$C_{\alpha f} = 62$
$C_{\alpha r} = 71$
$C_{\gamma f} = 7.7$
$C_{\gamma r} = 11.9$
$\frac{u^2}{GR} = .055, .219 \quad \phi = 3°, 12°$

$$\delta_{5MPH} = 57.3\left(\frac{5}{30}\right) + \left[\frac{400}{62} - \frac{600}{71}\right].055 + \left[\frac{11.9}{71} - \frac{7.7}{62}\right]3$$
$$= 9.54 + [6.47 - 8.45].055 + [.168 - .124]3$$
$$= 9.54 + [-1.98].055 + [.044]3$$
$$= 9.54 - .11 + .132 = 9.562°$$

$$\delta_{10MPH} = 9.54 + [-1.98].219 + [.044]12°$$
$$= 9.54 - .433 + .528$$
$$= 9.54 + .095 = 9.635°$$

IN THIS CASE, TIRE CAMBER PROPERTIES PROVIDE A SLIGHT UNDERSTEER INFLUENCE BUT THE VEHICLE APPEARS TO BE VIRTUALLY NEUTRAL STEER FOR THIS SITUATION.

CASE #3 - ARTICULATED TRICYCLE, ASSUMED DATA ON 2.80-4 TIRES

ASSUME: 50 LB VEHICLE AND 200 LB RIDER
CAMBER STIFFNESS = 15% CORNERING STIFFNESS
CORNERING STIFFNESS = 25 LB/DEG @ 110 LB.
16 LB/DEG @ 50 LB.

BASED ON LIMITED LIGHT LOAD SLIP ANGLE DATA TAKEN FROM A SIMILAR TIRE IN 1970.

$W_f = 30$ LB
$W_r = 220$ LB.
$\ell = 3$ ft
$R = 30$ ft
$u = 7.3$ ft/sec, $14.7$ ft/sec
$C_{\alpha f} = 16$ LB/DEG
$C_{\alpha r} = 25$ LB/DEG
$C_{\gamma f} = 2.4$ LB/DEG
$C_{\gamma r} = 3.8$ LB/DEG

$$\frac{u^2}{gR} = .055, .219$$

$$\delta_{5MPH} = 57.3\left(\frac{3}{30}\right) + \left[\frac{30}{16} - \frac{220}{50}\right].055 + \left[\frac{3.8}{25} - \frac{2.4}{16}\right]3^\circ$$

$$= 5.73 + \left[1.88 - 4.40\right].055$$

$$= 5.73 + \left[-2.52\right].055 = 5.73 - .14 = 5.59^\circ$$

$$\delta_{10MPH} = 5.73 + \left[-2.52\right].219 = 5.73 - .55 = 5.18^\circ$$

IF THE ABOVE DATA IS AT ALL VALID, THE VEHICLE MAY HAVE ABOUT 2.5% OF OVERSTEER BUT THE LARGE ACKERMANN CONTRIBUTION MINIMIZES THE INFLUENCE ON STEER ANGLE.

CASE #4 - SMALL CAR

ASSUME: 2000 LB VEHICLE AND PASSENGER LOAD. FOR THIS CASE, EQUATION (19) CAN BE WRITTEN IN TERMS OF TOTAL FRONT AND REAR CORNERING COMPLIANCES AS SHOWN IN ENGINEERING PUBLICATION 2771 (CORNERING COMPLIANCE CONCEPT). THIS BECOMES THEN

$$\delta = 57.3 \frac{l}{R} + \left[ D_f - D_R \right] \frac{u^2}{GR}$$ WHERE $\delta$ IS FRONT

WHEEL ANGLE WHICH CAN BE REPLACED BY

$\frac{\delta_{SW}}{SR}$ FOR STEERING WHEEL ANGLE.

CARS OF THIS CLASS ARE, TYPICALLY $D_F = 6°/G$

$D_R = 4°/G$

ASSUME: $l = 7$ FT
$R = 30$ FT
$u = 7.3$ ft/sec, $14.7$ ft/sec (5 MPH, 10 MPH)

$$\delta_5 = 57.3 \frac{(7)}{30} + \left[ 6 - 4 \right] .055 = 13.4 + .11 = 13.51°$$

$$\delta_{10} = 13.4 + \left[ 2 \right] .219 = 13.4 + .438 = 13.838°$$

STEERING WHEEL ANGLES WOULD BE ABOUT 20 X THAT OR

$$\underset{5MPH}{\delta_{SW}} = 261°, \quad \underset{10MPH}{\delta_{SW}} = 276°$$

CONSIDER NOW A HIGHER SPEED SITUATION FOR THE FOUR CLASSES OF VEHICLE CONSIDERED HERE.

$u$ @ 20 MPH = 29 FT/SEC $\qquad \phi = \tan^{-1} .358 = 20^\circ$

$R = 75$ ft

$$\frac{u^2}{GR} = \frac{864}{32.2(75)} = .358$$ { MAY NOT VIOLATE TIRE LINEARITY ASSUMPTIONS BADLY.

CASE #1 - BICYCLE - $\delta_{20} = 57.3\left(\frac{4}{75}\right) + (-2.19).358 = 3.06 - .78 = 2.28^\circ$

CASE #2 - MOTORCYCLE - $\delta_{20} = 57.3\left(\frac{5}{75}\right) + (-1.98).358 + [.044)20^\circ$

$= 3.82 - .71 + .88 \cong 4^\circ$

CASE #3 - ARTICULATED TRICYCLE

$$\delta_{20} = 57.3\left(\frac{3}{75}\right) + (-2.52).358 =$$

$$= 2.3 - .9 = 1.4^\circ$$

CASE #4 - SMALL CAR

$$\delta_{20} = \frac{57.3(7)}{75} + 2(.358) = 5.35 + .716$$

$= 6.066^\circ$ FRONT WHEEL ANGLE.

$= 121^\circ$ STEERING WHEEL ANGLE

## CRITICAL SPEED ANALYSIS

SINCE SEVERAL OF THE VEHICLES CONSIDERED HERE APPEAR TO HAVE SOME LINEAR RANGE OVERSTEER, (SUBJECT TO THE ADEQUACY OF LIMITED TIRE DATA) THE ISSUE OF CRITICAL SPEED SHOULD BE EXAMINED. EQUATION (19) IMPLIES THAT CRITICAL SPEED (SPEED FOR INFINITE GAIN) WILL BE A FUNCTION OF LATERAL ACCELERATION DUE TO THE $\tan^{-1}\frac{u^2}{GR}$ TERM. TO SIMPLIFY CALCULATIONS FOR THE BICYCLE AND ARTICULATED TRICYCLE, WE WILL NEGLECT THE CAMBER OPERATION TERM SINCE LIMITED TIRE DATA WILL NOT PERMIT ITS EVALUATION. SO EQUATION (19) BECOMES

$$\delta = 57.3\frac{l}{R} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right]\frac{u^2}{GR}$$

$$\frac{\delta}{\frac{u^2}{GR}} = 57.3\frac{lG}{u^2} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right]$$

OR

$$\frac{u^2/GR}{\delta} = \frac{1}{57.3\frac{lG}{u^2} + \left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right]}$$

THIS IS VERY LARGE WHEN $\left|\frac{W_r}{2C_{\alpha r}}\right| > \left|\frac{W_f}{C_{\alpha f}}\right|$

AND:

$$57.3\frac{lG}{u^2} = \left|\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right|$$

OR:

$$\mu = \sqrt{\frac{57.3\,\ell G}{\left|\frac{W_f}{C_{\alpha f}} - \frac{W_r}{2C_{\alpha r}}\right|}}$$

FOR THE BICYCLE (CASE #1)

$$\mu_{CRITICAL} = \sqrt{\frac{57.3(4)(32.2)}{2.19}} = \sqrt{3370} = 58\ \frac{ft}{SEC}$$

$$= 40\ MPH.$$

FOR THE ARTICULATED TRICYCLE (CASE #3)

$$\mu_{CRITICAL} = \sqrt{\frac{57.3(3)\,32.2}{2.52}} = \sqrt{2200} = 47\ ft/sec$$

$$= 32\ MPH.$$

FOR 16 INCH WHEEL BASE;

$$\mu_{CRITICAL} = \sqrt{\frac{57.3(1.33)(32.2)}{2.19}} = \sqrt{1122} = 33\ ft/sec$$

$$= 23\ MPH,$$

THESE VERY APPROXIMATE CALCULATIONS CAN BE SUMMARIZED IN THE FOLLOWING TABLE.

| VEHICLE | UNDER STEER OVERSTEER | CRITICAL SPEED | .06 G STEER 5 MPH 30 FT RAD. | .22 G STEER 10 MPH 30 FT RAD. | .36 G STEER 20 MPH 75 FT RAD. |
|---|---|---|---|---|---|
| BICYCLE | -2.2% | 40 MPH | 7.5° | 7.2° | 2.3° |
| MOTORCYCLE | 0.1% | — | 9.6° | 9.6° | 4.0° |
| ARTICULATED TRICYCLE | -2.5% | 32 MPH | 5.6° | 5.2° | 1.4° |
| SMALL CAR | 2.0% | — | 261° (13.5) | 276° (13.8°) | 121° — STEERING WHEEL (6.1°) — FRONT WHEEL |

SOME VERY TENTATIVE SPECULATIONS CAN BE DRAWN FROM THIS:

1. BELOW 10 MPH, ACKERMANN GEOMETRY CONSIDERATIONS APPEAR TO DOMINATE STEERING SENSITIVITY IN THE LOW LATERAL ACCELERATION REGION. STEERING SENSITIVITY IS CONTROLLED BY WHEELBASE. VEHICLES WILL PROBABLY APPEAR TO BE NEUTRAL STEER WITH CONSTANT STEER ANGLE AND INCREASING CAMBER AS SPEED INCREASES ON A CONSTANT RADIUS.

2. AT 20 MPH, WEIGHT DISTRIBUTION AND TIRE PROPERTIES ARE BEGINNING TO INFLUENCE STEERING SENSITIVITY SIGNIFICANTLY. THIS IS PARTICULARLY TRUE OF THE SHORT WHEELBASE ARTICULATED TRICYCLE WITH A HEAVY RIDER.

3. IT IS POSSIBLE THAT CRITICAL SPEED FOR THE ARTICULATED TRICYCLE MAY BE ABOUT 50% ABOVE DESIGN TOP SPEED. REDUCING WHEEL BASE SHOULD MOVE CRITICAL SPEED TOWARD TOP SPEED, BUT IT CHANGES AS THE SQUARE ROOT OF WHEELBASE.

4. TOTAL CORNERING COMPLIANCES FOR SMALL CARS, BIKES, AND MOTORCYCLES APPEAR ROUGHLY COMPARABLE IN MAGNITUDE (IN THE RANGE OF 4 - 6 DEG/G). THIS IMPLIES THAT THEY SHOULD OPERATE AT SIMILAR SLIP ANGLES. THE ARTICULATED TRICYCLE MAY HAVE RATHER LOWER CORNERING COMPLIANCE WITH THE REAR APPROACHING THAT OF A SMALL CAR AND THE FRONT MUCH LOWER.

GUS GUSAKOV:

TEST PROGRAM FOR CARVED TREAD

3.14-4 GOODYEAR TIRE

LOAD-SLIP ANGLE-INCLINATION ANGLE RELATIONSHIP

ROAD - DRY - CONVENIENT LOW SPEED

WET - (.010 WATER) - 20 MPH.

LOADS:- 50, 100, 150 LB.

INCLINATION ANGLE: 0, 10°, 20°, 30°, 40°, 50°

SLIP ANGLE: 0, ±1°, ±2°, 4°, 6°, 10°

SLIP - INCLINATION SIGNS ARE THOSE FOR MOTOR CYCLE OPERATION.

IF CONVENIENT, SUPPLY RAW DATA CARPET PLOTS OF SLIP-CAMBER ANGLE WITH A SURFACE FOR EACH LOAD CONDITION.

INFLATION - 20 PSI

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4-11-75

# SUMMARY

## VEHICLE DIRECTIONAL PERFORMANCE

## BASED ON CALSPAN TIRE DATA

| | |
|---|---|
| LOCKED WHEEL DECELERATION CAPABILITY (CG DEPENDENT) - DRY | 0.45 G |
| WET | 0.40 G's |
| MAXIMUM LATERAL ACCELERATION CAPABILITY - DRY | 1.00 G's |
| WET | 0.90 G's |
| LINEAR RANGE UNDERSTEER | 2.0 °/G |
| ROLLING RESISTANCE COEFFICIENT (20 PSI) | .019 |
| STEER CHARACTERISTIC AT CONTROL LIMIT | UNDERSTEER |

TIRE: GOODYEAR
3.14-4
NARROW RIB TREAD
20 PSI

WET = .010" WATER
3M BELT MATERIAL

ASSUMPTIONS:

WEIGHT DISTRIBUTION 50 LB-FRONT, 200 LB-REAR
40 INCH CG HEIGHT

CG REMAINS IN THE PLANE OF SYMMETRY AND WEIGHT DISTRIBUTION IS CONSTANT FOR BRAKING AND CORNERING.

RAKE ANGLE PHENOMENA ARE NEGLECTED. (STEADY STATE DATA WERE REANALYZED INCLUDING THE INTERACTION BETWEEN RAKE ANGLE AND FRONT TIRE CAMBER. THIS EFFECT MAKES FRONT TIRE CAMBER ABOUT 2° GREATER THAN REAR FOR A 30 FT RADIUS. THE EFFECT IS NOT SIGNIFICANT.)

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4-11-75

# TIRE DATA ANALYSIS SUMMARY

## INTRODUCTION

CALSPAN CORPORATION HAS COMPLETED EXTENSIVE FORCE AND MOMENT TESTING ON TWO PRETEST TIRES FOR THE PROJECT 300 VEHICLE. THE INITIAL 2.80-4 GROUND DOWN PRODUCTION MOLD TIRE WAS TESTED TO ESTABLISH MACHINE CAPABILITY AND TESTING TECHNIQUE. MORE EXTENSIVE TESTS WERE THEN RUN ON A 3.14-4 SPECIAL MOLD TIRE DEVELOPED FOR PROTOTYPE USAGE. BOTH SETS OF DATA WERE PLOTTED AND ANALYZED BUT THE 3.14-4 DATA ARE PROBABLY MORE APPLICABLE. REMOVAL OF TREAD RUBBER FROM THE 2.80-4 APPEARED TO PRODUCE CHARACTERISTICS NOT REPRESENTATIVE OF A PRACTICAL TIRE.

THREE SETS OF TEST DATA WERE OBTAINED TO EVALUATE THE POWERLOSS, BRAKING, AND HANDLING ASPECTS OF VEHICLE PERFORMANCE. THESE DATA WERE APPLIED TO SOME RELATIVELY SIMPLE CALCULATIONS OF VEHICLE PERFORMANCE. THE RESULTS ARE SUMMARIZED ON THE COVER PAGE OF THIS REPORT.

THE RESULTS OF THIS TESTING AND ANALYSIS MUST BE CAREFULLY QUALIFIED.

1. THE TIRF MACHINE AT CALSPAN WAS NOT INTENDED FOR MEASUREMENTS OF ~~T~~ VERY SMALL TIRES WHERE MANY FORCES OF INTEREST ARE MUCH LESS THAN 200 LB. PLOTS OF RAW DATA SHOW SOME VARIABILITY AND ANOMALIES THOUGHT TO BE PRODUCED MORE

BY THE TEST TECHNIQUE THAN THE TIRES. JUDGEMENT AND EXPERIENCE WERE USED TO ESTIMATE THE MOST PROBABLE TIRE PROPERTIES FROM THE RAW DATA. DIFFERENT TEST TECHNIQUES AND STATISTICAL SURFACE FITTING OF RAW DATA COULD BE USED TO PRODUCE MORE ACCURATE RESULTS; GIVEN SUFFICIENT TIME AND RESOURCES.

2. THE BRAKING AND DIRECTIONAL PERFORMANCE OF ANY VEHICLE DEPENDS STRONGLY ON THE LOCATION OF THE GRAVITY CENTER. FOR THE PROJECT 300 VEHICLE, THE RIDER HAS THE OPPORTUNITY TO MAKE LARGE ADJUSTMENTS IN CG LOCATION DEPENDING ON HIS CHOICE OF CONTROL STRATEGY. SOME ASSUMPTIONS ON CG LOCATION ARE REQUIRED FOR TIRE DATA ANALYSIS AND THEIR VALIDITY WILL DEPEND ON RIDER STRATEGIES THAT ARE PRESENTLY UNKNOWN. IF TIRE DATA WERE REDUCED TO A MATHEMATICAL MODEL, SOME SIMPLE COMPUTER PROGRAMS COULD BE WRITTEN TO PRODUCE PERFORMANCE MAPS FOR THE MANY POSSIBLE APPROACHES TO VEHICLE OPERATION.

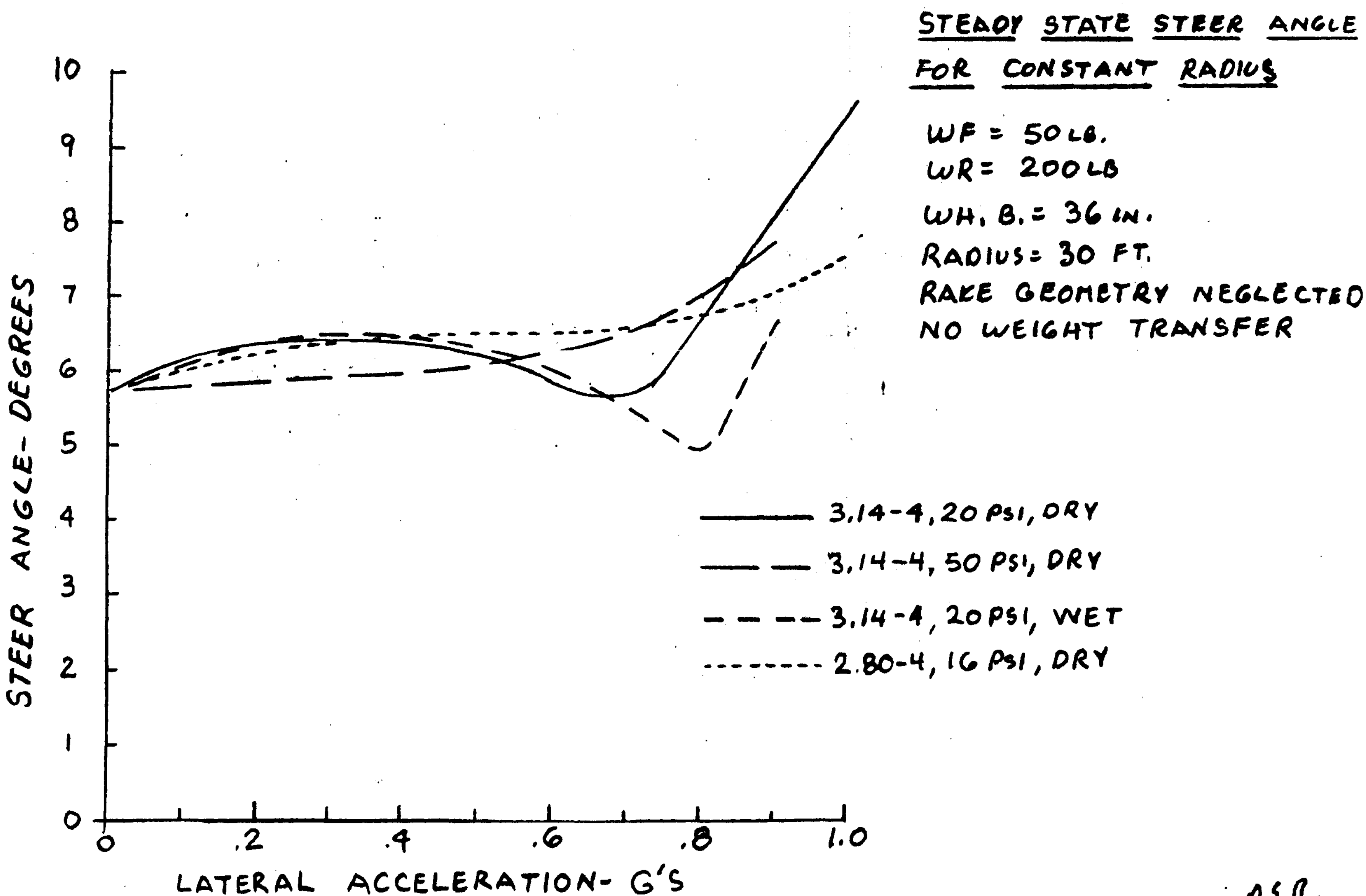
POWER LOSS AND LOCKED WHEEL TRACTION DATA ARE EASILY PRESENTED IN NUMERICAL OR GRAPHICAL FORM. HOWEVER, LATERAL FORCE DATA ARE DIFFICULT TO SUMMARIZE SINCE SLIP ANGLE, CAMBER ANGLE, AND NORMAL LOAD EACH EXERT A MAJOR INFLUENCE ON THE DEVELOPMENT OF LATERAL FORCE. ANALYSIS OF PASSENGER TIRE DATA HAS TRADITIONALLY BEEN APPROACHED WITH PLOTS OF LATERAL FORCE AS A FUNCTION OF

SLIP ANGLE AND NORMAL LOAD. SINCE PASSENGER CAR DIRECTIONAL PERFORMANCE IS INFLUENCED BY LATERAL WEIGHT TRANSFER, THESE PLOTS HAVE PROVIDED USEFUL INSIGHT TO THE TIRE'S CONTRIBUTION TO VEHICLE HANDLING. FOR THE PROJECT 300 VEHICLE, HANDLING IS THOUGHT TO BE INFLUENCED MORE BY SLIP AND CAMBER ANGLE PHENOMENA AND LESS BY WEIGHT TRANSFER, THEREFORE, CALSPAN DATA WERE PLOTTED FOR THREE CONSTANT LOAD CONDITIONS (50LB, 100LB, 150LB) IN THE FORM OF SURFACES REPRESENTING THE INFLUENCE OF SLIP AND CAMBER ANGLE ON LATERAL FORCE. MANY OTHER PLOTTING FORMATS ARE, OF COURSE, POSSIBLE. THE ONE CHOSEN HERE PROVED CONVENIENT FOR CALCULATION OF SKID PAD TYPE DATA.

CONCLUSIONS

SUBJECT TO THE ABOVE QUALIFICATIONS, THE FOLLOWING CONCLUSIONS ARE EVIDENT FROM THE ANALYSIS DONE SO FAR:

1. THE PROJECT 300 VEHICLE, EQUIPPED WITH 3.14-4 GOODYEAR TIRES AT 20 PSI INFLATION SHOULD SHOW A SMALL DEGREE OF LOW LATERAL ACCELERATION UNDERSTEER (1-2°/G) WITH AN UNDERSTEER TYPE CONTROL LIMIT FOR BOTH DRY AND WET CONDITIONS. CORNERING IN THE RANGE OF 1.0 G APPEARS POSSIBLE ON A SURFACE EQUIVALENT TO THE TIRE BELT. WET CORNERING AT 0.9 G MAY BE POSSIBLE WITH MINIMAL WATER ON A GOOD SURFACE. THESE STEADY STATE DATA ARE SHOWN ON THE FOLLOWING GRAPH. INCLUSION OF RAKE GEOMETRY EFFECTS ON FRONT TIRE CAMBER DOES NOT CHANGE THESE PLOTS SIGNIFICANTLY.


STEADY STATE STEER ANGLE
FOR CONSTANT RADIUS
WF = 50 LB.
WR = 200 LB
WH. B. = 36 IN.
RADIUS = 30 FT.
RAKE GEOMETRY NEGLECTED
NO WEIGHT TRANSFER
STEER ANGLE- DEGREES
0
1
2
3
4
5
6
7
8
9
10
0
.2
.4
.6
.8
1.0
LATERAL ACCELERATION- G'S
3.14-4, 20 PSI, DRY
3.14-4, 50 PSI, DRY
3.14-4, 20 PSI, WET
2.80-4, 16 PSI, DRY

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4-10-75

2. SMALL CHANGES IN TIRE INFLATION (4 PSI) PRODUCE INSIGNIFICANT CHANGES IN LATERAL FORCE PERFORMANCE FOR THESE TIRES. LARGER CHANGES (20 PSI - 50 PSI) HAVE A MEASURABLE INFLUENCE ON PERFORMANCE WITH TRENDS UNLIKE PASSENGER TIRES. THE <u>LOWER</u> PRESSURES PRODUCE <u>HIGHER</u> CORNERING AND CAMBER STIFFNESSES TOGETHER WITH <u>HIGHER</u> MAXIMUM FORCE CAPABILITY. ANALYSIS OF 50 PSI DATA IMPLIED THAT THE VEHICLE MIGHT HAVE A VERY SLIGHT DEGREE OF OVERSTEER FOR THIS CONDITION.

3. LINEAR RANGE OVER/UNDERSTEER IS DETERMINED BY FRONT AND REAR CORNERING AND CAMBER STIFFNESSES WHICH ARE, IN GENERAL, DIFFERENT DUE TO THE DIFFERENT LOAD CONDITIONS. THE RELATIONSHIP IS THE FOLLOWING:

$$K = \left[\frac{W_f}{C_{\alpha f}} - 56.7\frac{C_{\gamma f}}{C_{\alpha f}}\right] - \left[\frac{W_r}{2C_{\alpha r}} - 56.7\frac{C_{\gamma r}}{C_{\alpha r}}\right]$$

WHERE: $W_f, W_r$ = FRONT AND REAR WEIGHT

$C_{\alpha f}, C_{\alpha r}$ = FRONT AND REAR CORNERING STIFFNESS

$C_{\gamma f}, C_{\gamma r}$ = FRONT AND REAR CAMBER STIFFNESS

DUE TO THE LARGE MULTIPLICATIVE FACTOR (56.7) THE CAMBER STIFFNESS/CORNERING STIFFNESS RATIOS HAVE A SIGNIFICANT INFLUENCE ON TOTAL VEHICLE UNDERSTEER. NONE OF THE ABOVE FOUR TERMS CAN BE NEGLECTED.

4. THE IMPLICATIONS OF UNDERSTEER TO VEHICLE STEERING SENSITIVITY ARE SHOWN IN THE FOLLOWING GRAPH. UNDERSTEER IS NOT USUALLY SIGNIFICANT TO THE STEERING SENSITIVITY OF PASSENGER CARS IN THIS SPEED RANGE BUT THE SHORT WHEELBASE OF THE PROJECT 300 VEHICLE PRODUCES A DIFFERENT SITUATION. IT APPEARS THAT SIGNIFICANT REDUCTIONS IN STEERING SENSITIVITY, IN A DISPLACEMENT SENSE, ARE POSSIBLE BY INTRODUCING REAR ROLL UNDERSTEER OR, PERHAPS, TORSIONAL COMPLIANCE IN THE STEERING SHAFT TO PRODUCE LATERAL FORCE COMPLIANCE UNDERSTEER IN THE FRONT. THIS MIGHT BE INVESTIGATED AS A SUPPLEMENT TO THE CURRENT TECHNIQUE OF USING LARGE CASTER OFFSET TO PRODUCE HIGH STEERING RETURN FORCES. THE RELATIVE MERITS OF INCREASED FORCE GAIN VERSUS REDUCED DISPLACEMENT GAIN CAN ONLY BE ASSESSED SUBJECTIVELY.

5. ALIGNING TORQUE DATA WERE PLOTTED FOR A FRONT TIRE CONDITION (50 LB LOAD) TO DETERMINE WHETHER THIS ASPECT OF TIRE PERFORMANCE IS SIGNIFICANT TO STEERING FORCE GRADIENT. DATA ARE GENERALLY BELOW 15 IN-LB AND FORM NO CONSISTENT PATTERN. IT APPEARS THAT THE ALIGNING TORQUES ARE WITHIN THE NOISE LEVEL OF THE TIRF SYSTEM.

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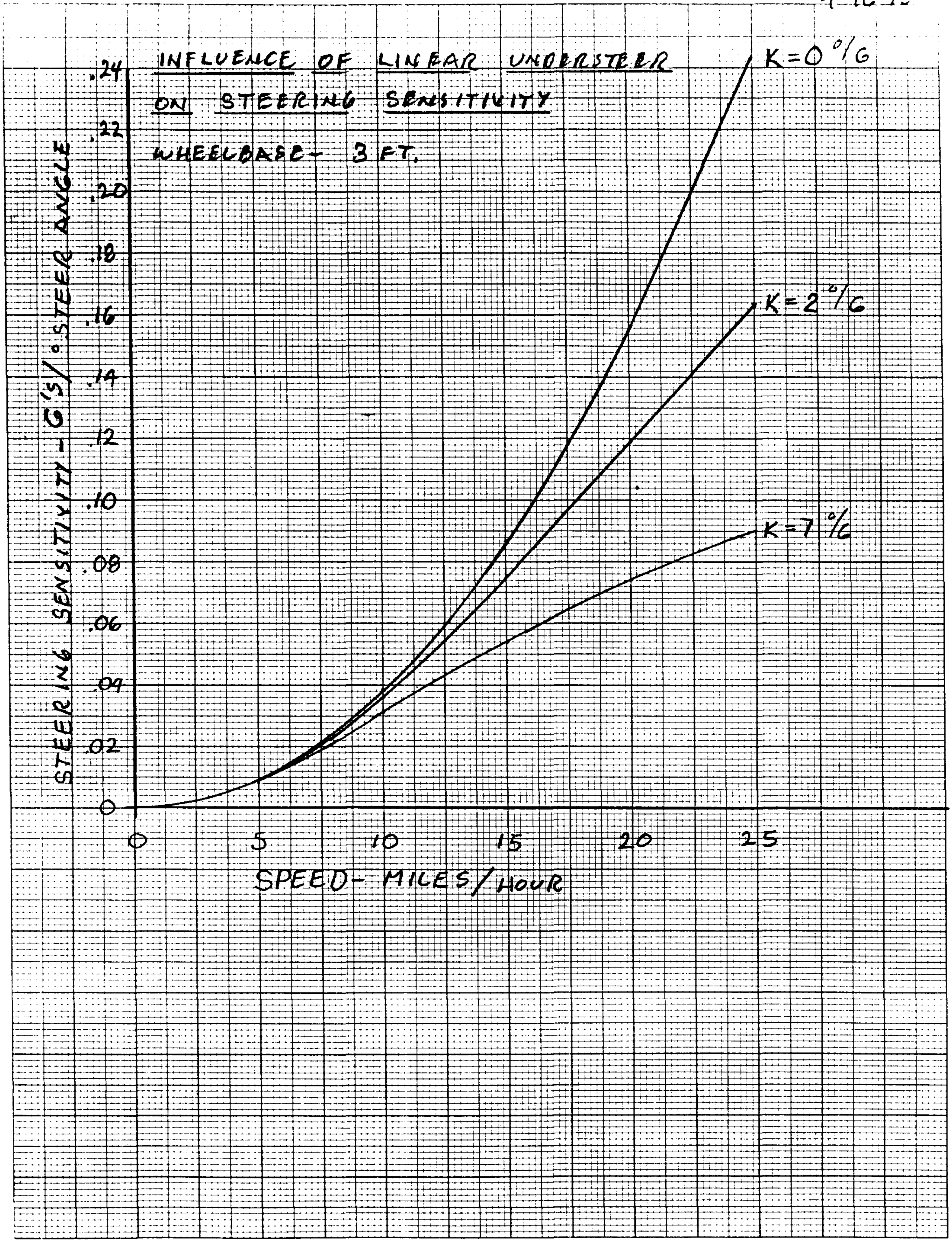
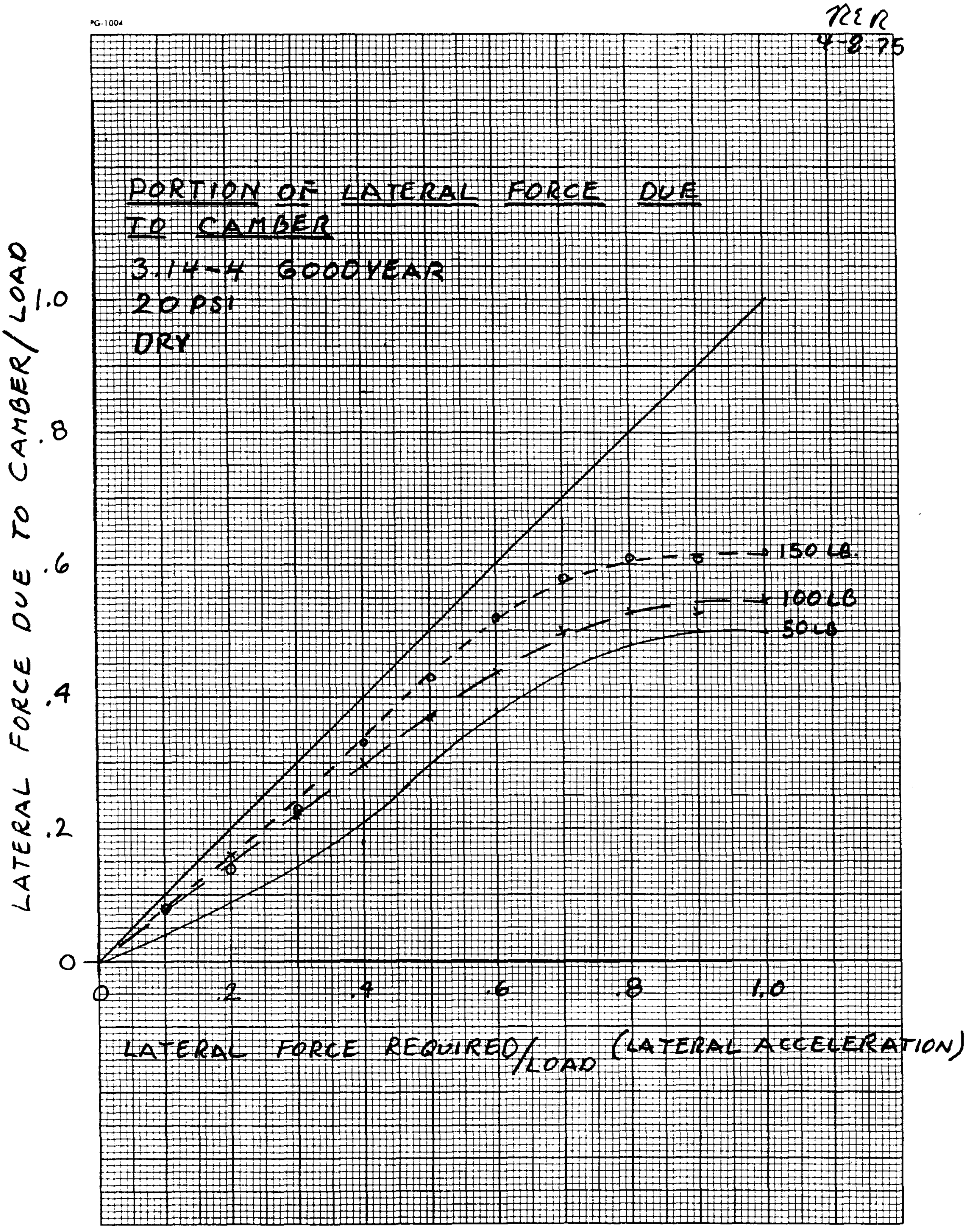
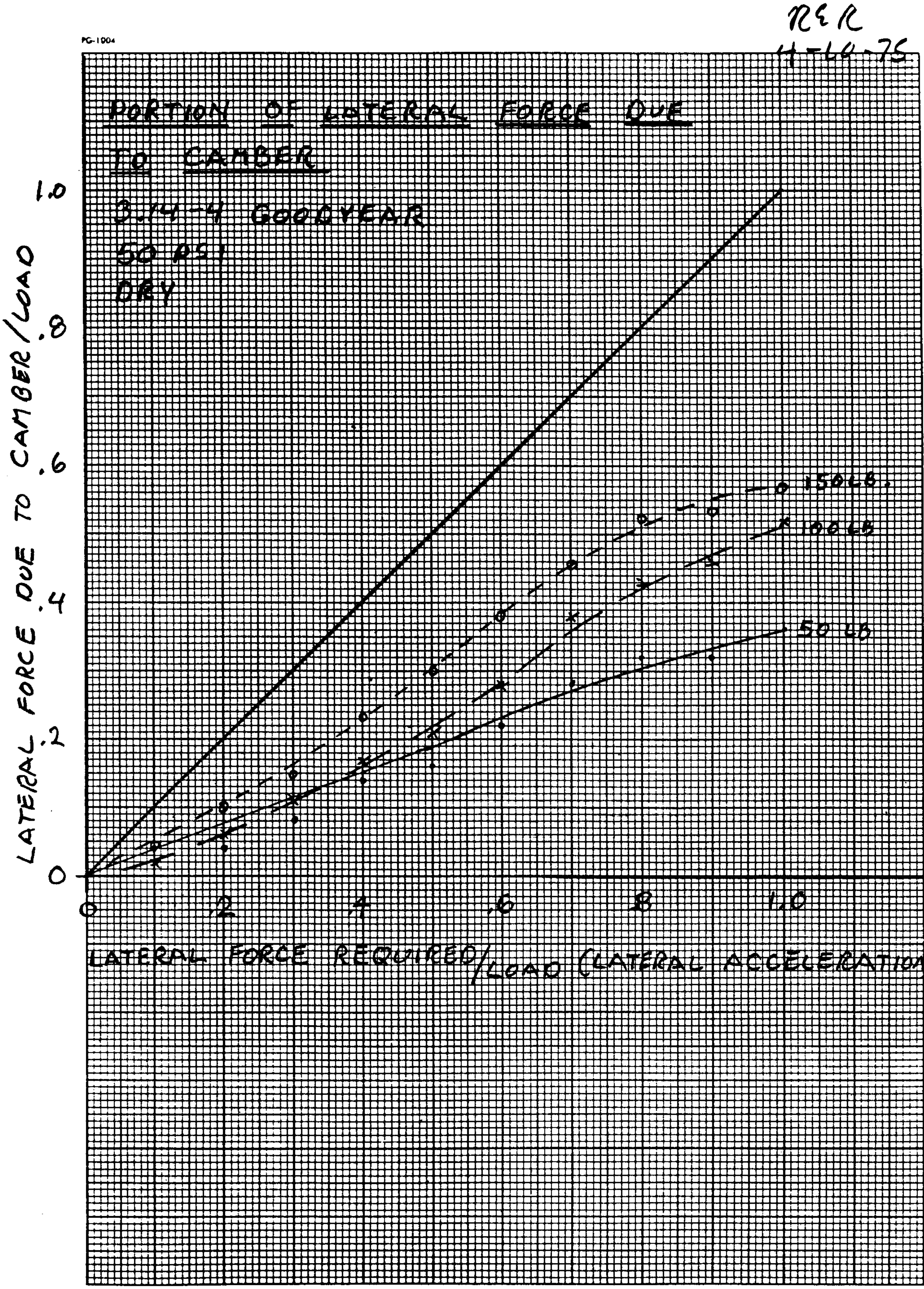

INFLUENCE OF LINEAR UNDERSTEER
ON STEERING SENSITIVITY
WHEELBASE- 3 FT.
K=0 °/G
K=2 °/G
K=7 °/G
STEERING SENSITIVITY - G'S/°STEER ANGLE
.24
.22
.20
.18
.16
.14
.12
.10
.08
.06
.04
.02
0
0
5
10
15
20
25
SPEED- MILES/HOUR

6. THE QUESTION OF THE PORTION OF LATERAL FORCE ARISING FROM CAMBER ANGLE AND SLIP ANGLE IS ADDRESSED BY THE FOLLOWING GRAPHS FOR THREE TIRE CONDITIONS. THE 45° DIAGONAL LINE REPRESENTS A CONDITION WHERE ALL OF THE REQUIRED CORNERING FORCE ARISES FROM CAMBER. DATA ABOVE THIS LINE IMPLIES THAT EXCESSIVE CAMBER FORCE IS PROVIDED AND NEGATIVE SLIP ANGLES ARE REQUIRED FOR EQUILIBRIUM. FOR THE 3.14-4 TIRE, CAMBER ANGLE IS RESPONSIBLE FOR SOMEWHAT LESS THAN THE REQUIRED LATERAL FORCE FOR ALL CONDITIONS. THEREFORE, SLIP ANGLE IS REQUIRED TO SUPPLEMENT THE FORCES DERIVED FROM CAMBER. MORE SLIP ANGLE IS REQUIRED FOR HIGHER LATERAL ACCELERATIONS, PARTICULARLY IN THE WET ROAD CASE. THE PREVIOUS GROUND 2.80-4 TIRE PRODUCED GREATER THAN THE REQUIRED LATERAL FORCE DUE TO CAMBER ANGLE.

7. THE WET ROAD LATERAL FORCE CARPET PLOTS SHOW A MARKED PEAK IN LATERAL FORCE DUE TO CAMBER (PLOTS INCLUDED AT THE END OF THESE NOTES) THE LINES OF CONSTANT SLIP ANGLE TEND TO PEAK BETWEEN 20° AND 30° CAMBER ANGLE EVEN THOUGH GREATER FORCES CAN BE ACHIEVED WITH INCREASED SLIP ANGLE. THE IMPLICATIONS OF THIS PHENOMENON TO WET ROAD HANDLING ARE NOT OBVIOUS.


PG-1004
RER
4-8-75
PORTION OF LATERAL FORCE DUE TO CAMBER
3.14-4 GOODYEAR
20 PSI
DRY
LATERAL FORCE DUE TO CAMBER/LOAD
LATERAL FORCE REQUIRED/LOAD (LATERAL ACCELERATION)
150 LB.
100 LB
50 LB
0
.2
.4
.6
.8
1.0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4-16-75
PG-1004
PORTION OF LATERAL FORCE DUE TO CAMBER
3.14-4 GOODYEAR
50 PSI
DRY
LATERAL FORCE DUE TO CAMBER/LOAD
1.0
.8
.6
.4
.2
0
150 LB.
100 LB
50 LB
0
.2
.4
.6
.8
1.0
LATERAL FORCE REQUIRED/LOAD (LATERAL ACCELERATION)

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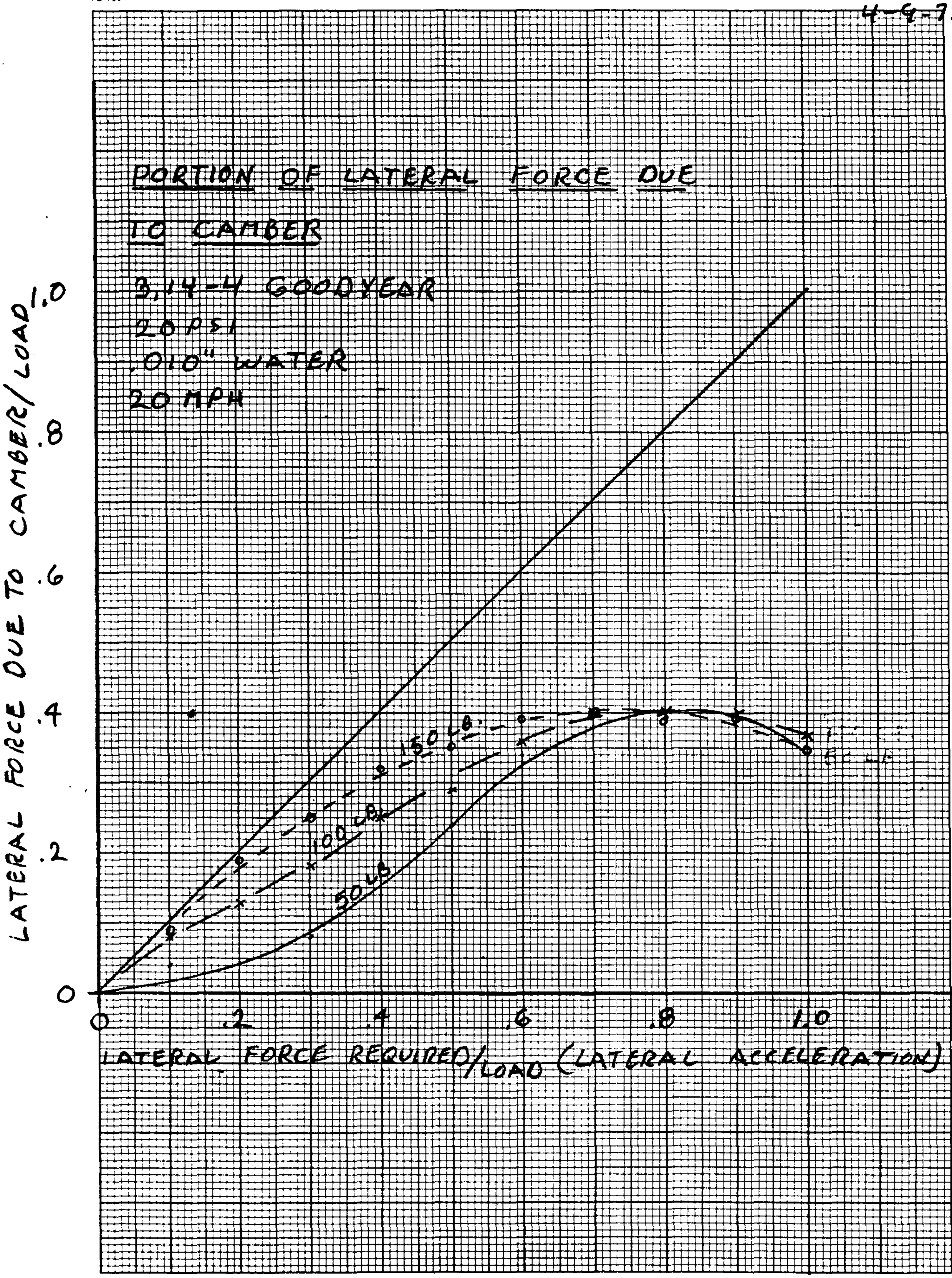
4-9-75


PORTION OF LATERAL FORCE DUE TO CAMBER
3.14-4 GOODYEAR
20 PSI
.010" WATER
20 MPH
LATERAL FORCE DUE TO CAMBER/LOAD
1.0
.8
.6
.4
.2
0
.2
.4
.6
.8
1.0
150 LB.
100 LB
50 LB
LATERAL FORCE REQUIRED/LOAD (LATERAL ACCELERATION)

8. IT IS INTERESTING TO NOTE THAT THE LOCKED WHEEL BRAKING TRACTION DATA ARE QUITE VARIABLE WITH RANGES IN OBSERVED FRICTION OF ABOUT 20% FOR THESE CLOSELY CONTROLLED CONDITIONS.

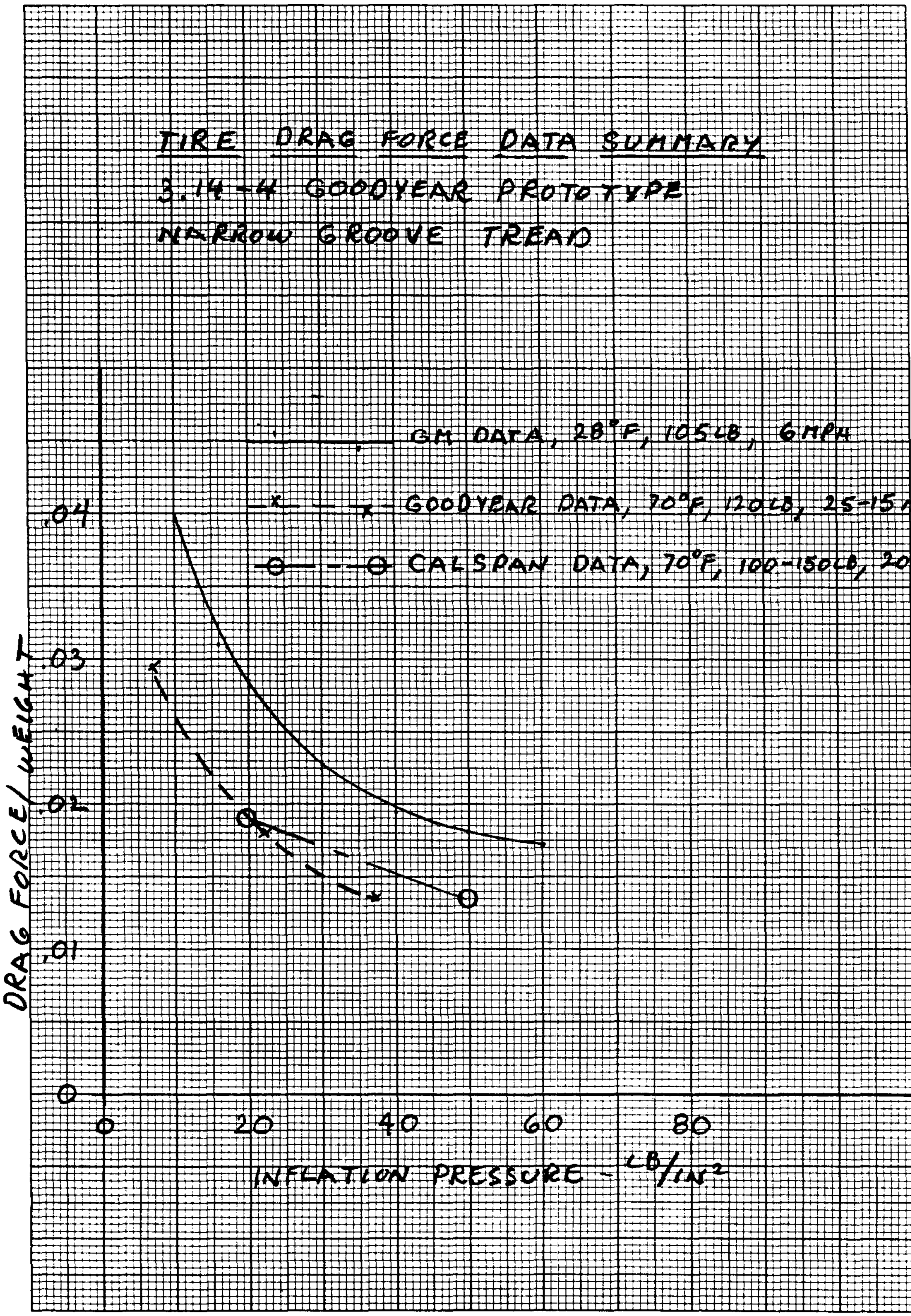
| | LOAD-LB. | |
|---|---|---|
| 20PSI | 50 | 150 |
| DRY | 1.26 | 1.12 |
| WET | .90 | .99 |

9. CALSPAN DATA ON THE FREE ROLLING DRAG FOR THE 3.14-4 TIRE WAS PRINTED WITH GREATER RESOLUTION THAN PREVIOUS DATA. THE LISTINGS SHOW SUBSTANTIAL SCATTER BUT IT IS POSSIBLE TO COMPUTE SOME AVERAGE VALUES THAT APPEAR TO BE RELEVANT.

THESE DATA ARE COMPARED WITH GM AND GOODYEAR DATA FOR SIMILAR TIRES IN THE FOLLOWING GRAPH. THE TWO DATA SETS TAKEN AT ROOM TEMPERATURE SHOW GOOD AGREEMENT.

REP
4-8-75

PG-1004



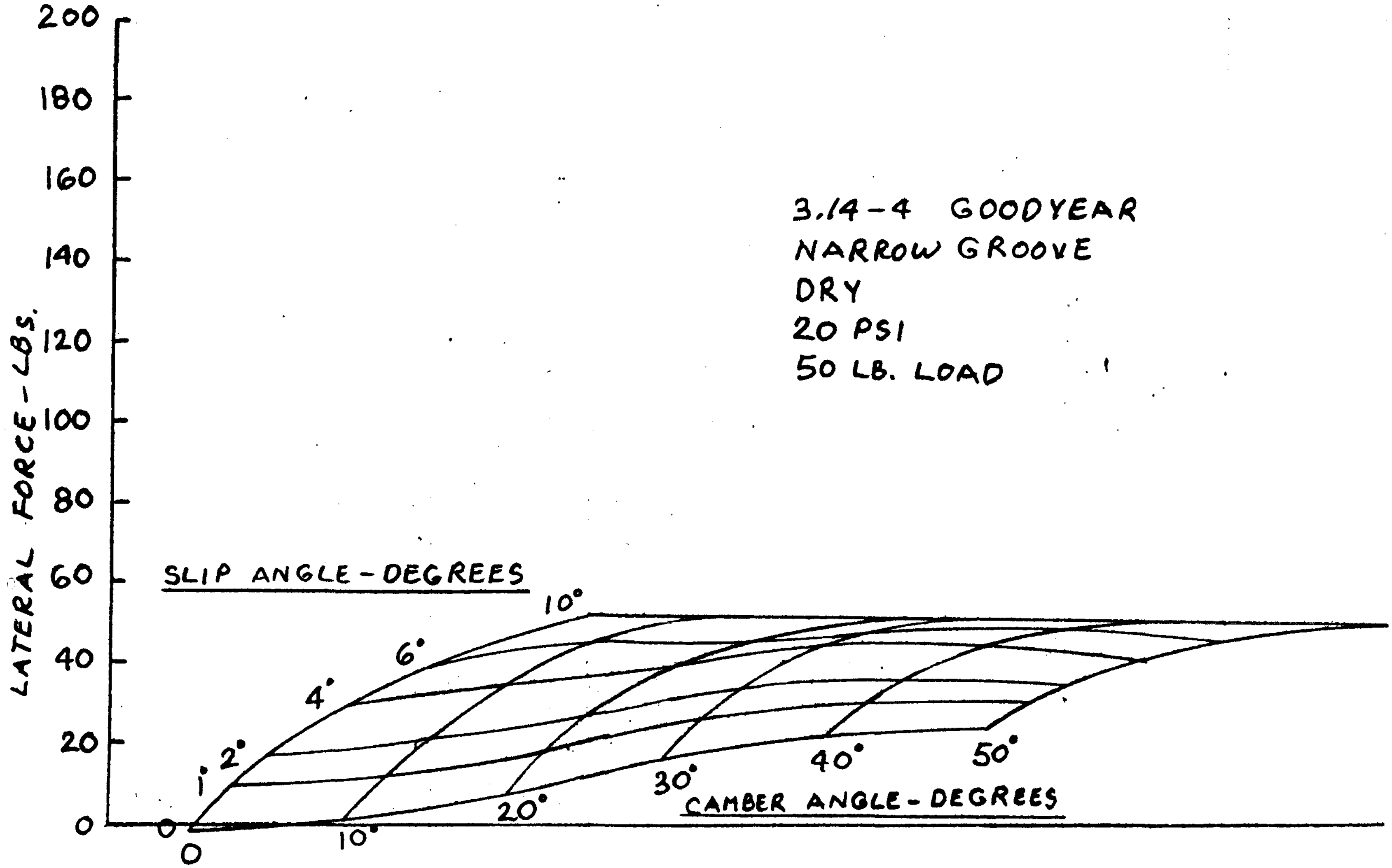
LATERAL FORCE CARPET PLOTS

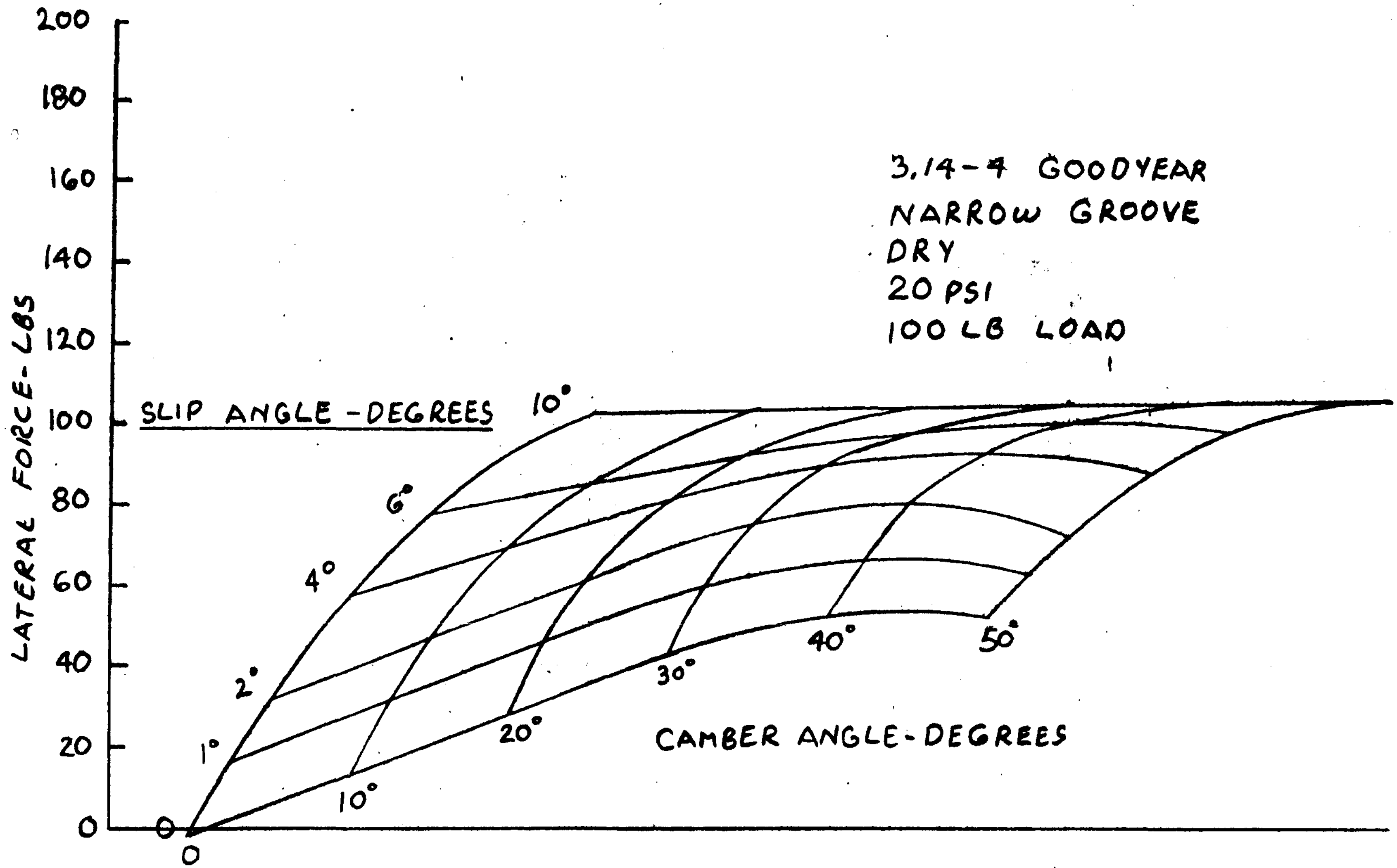
GOODYEAR 3.14-4

2 INCH RIM WIDTH

NARROW GROOVED TREAD

CONSTRUCTION # 8011


3.14-4 GOODYEAR
NARROW GROOVE
DRY
20 PSI
50 LB. LOAD
LATERAL FORCE - LBS.
200
180
160
140
120
100
80
60
40
20
0
SLIP ANGLE - DEGREES
0
1°
2°
4°
6°
10°
CAMBER ANGLE - DEGREES
0
10°
20°
30°
40°
50°

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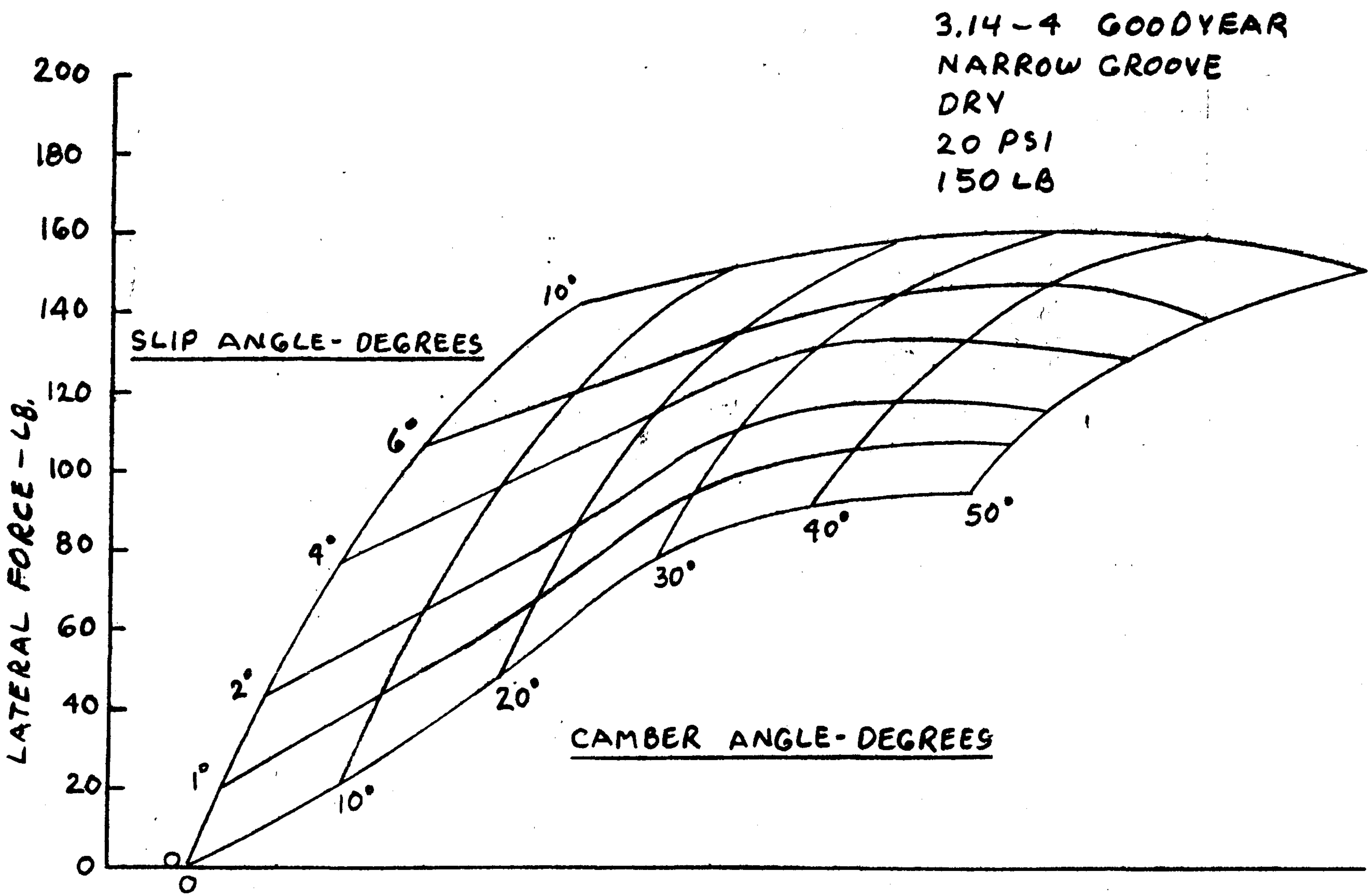
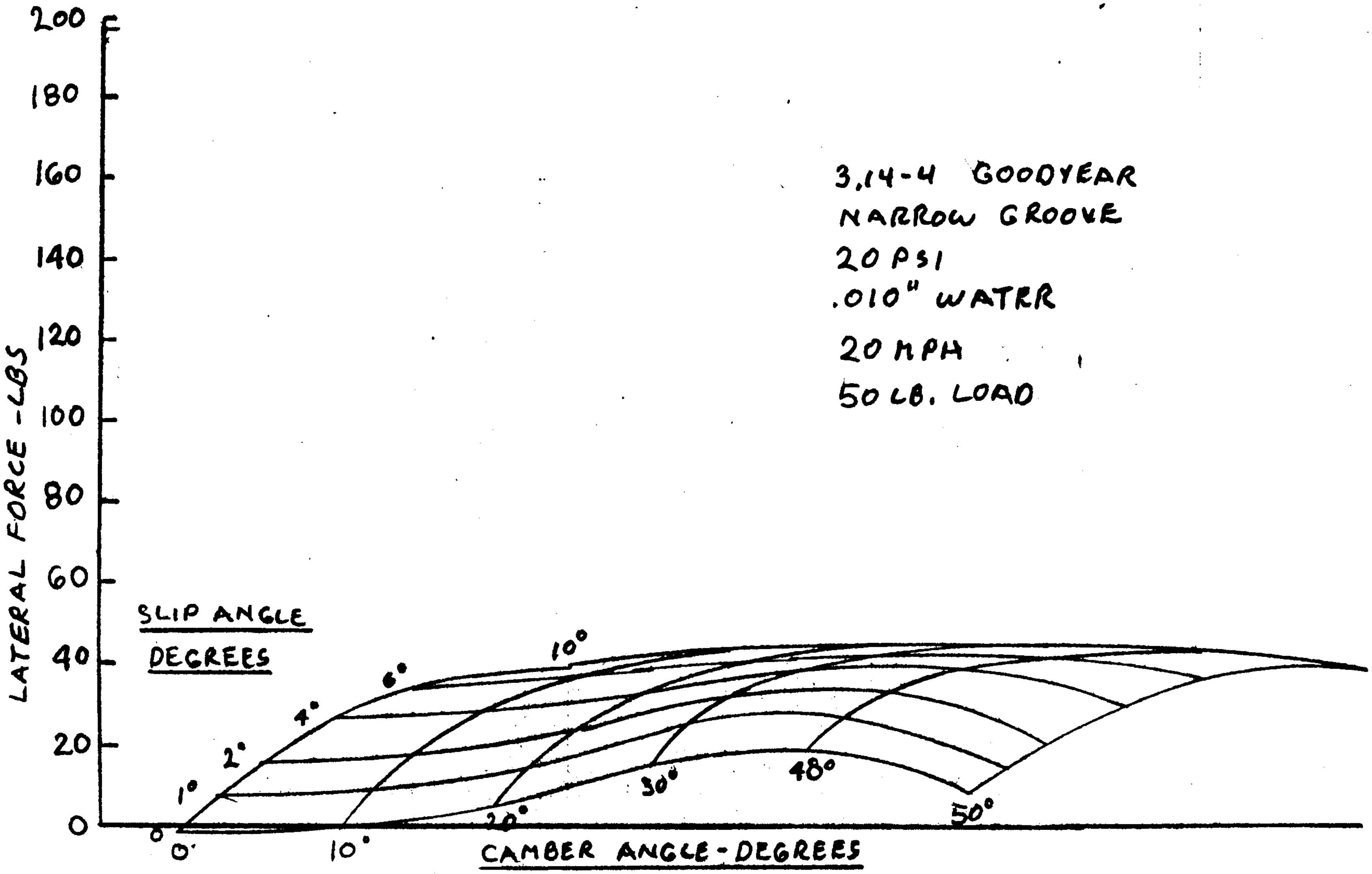
| TITLE | DATE | CHK. | REVISIONS—WAS | LET. | DATE |
|---|---|---|---|---|---|
| | BY | | | | |
| REF. DWG. NO. | PROJECT | | | | |
| SHEET OF / REQUESTED BY | DEPT. | | | | |


3.14-4 GOODYEAR
NARROW GROOVE
DRY
20 PSI
150 LB
SLIP ANGLE-DEGREES
CAMBER ANGLE-DEGREES
LATERAL FORCE - LB.
200
180
160
140
120
100
80
60
40
20
0
0
1°
2°
4°
6°
10°
10°
20°
30°
40°
50°

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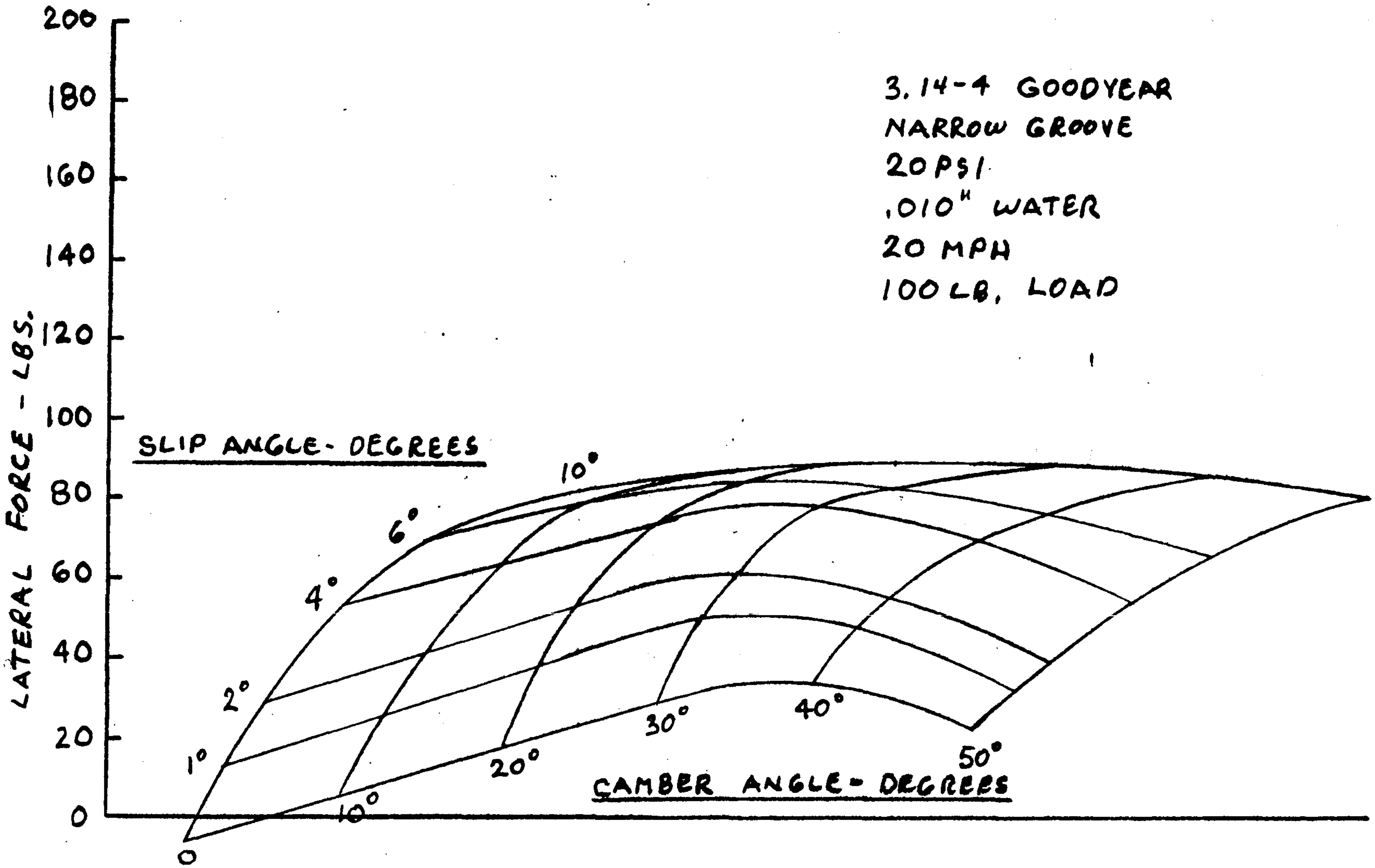
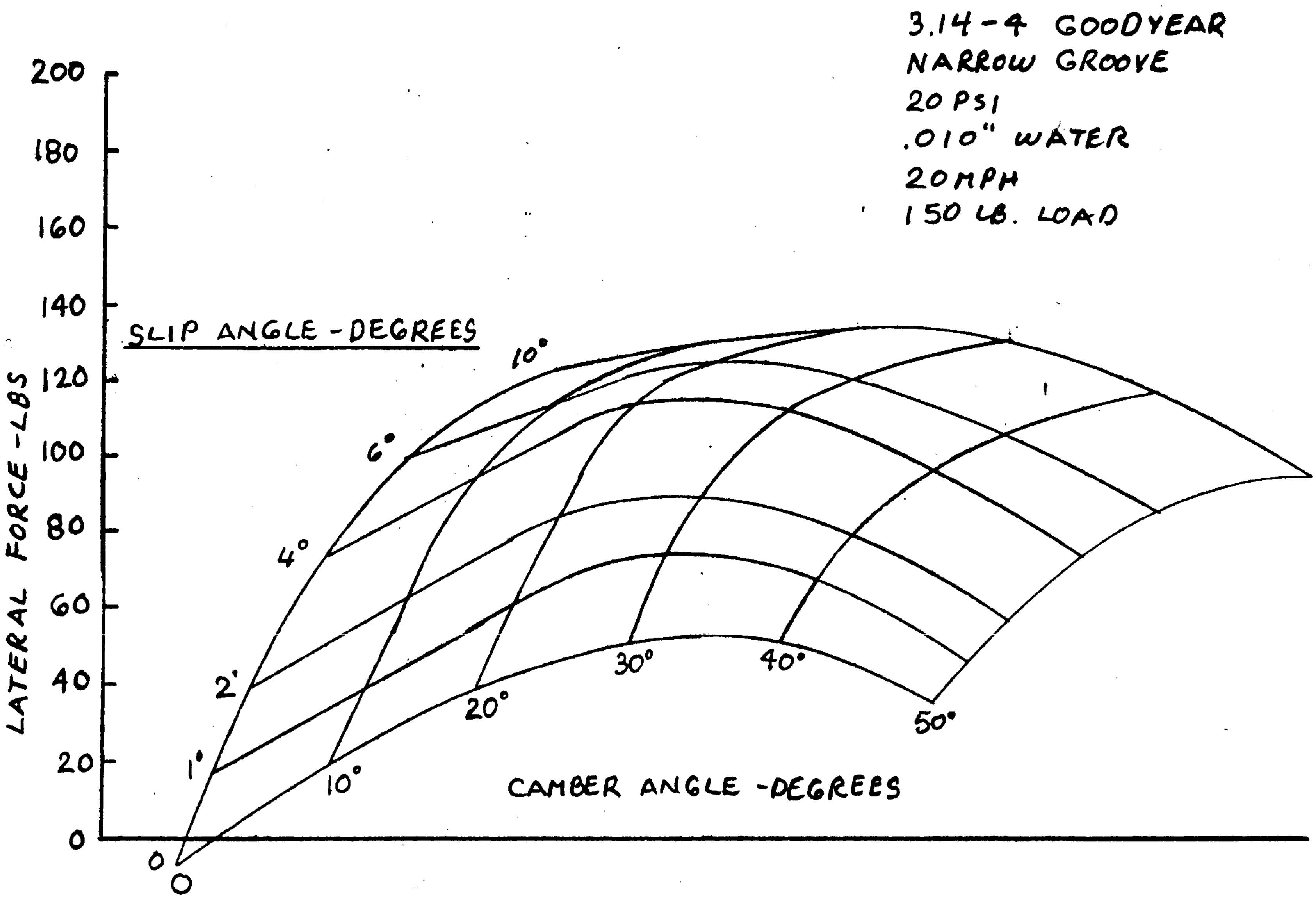
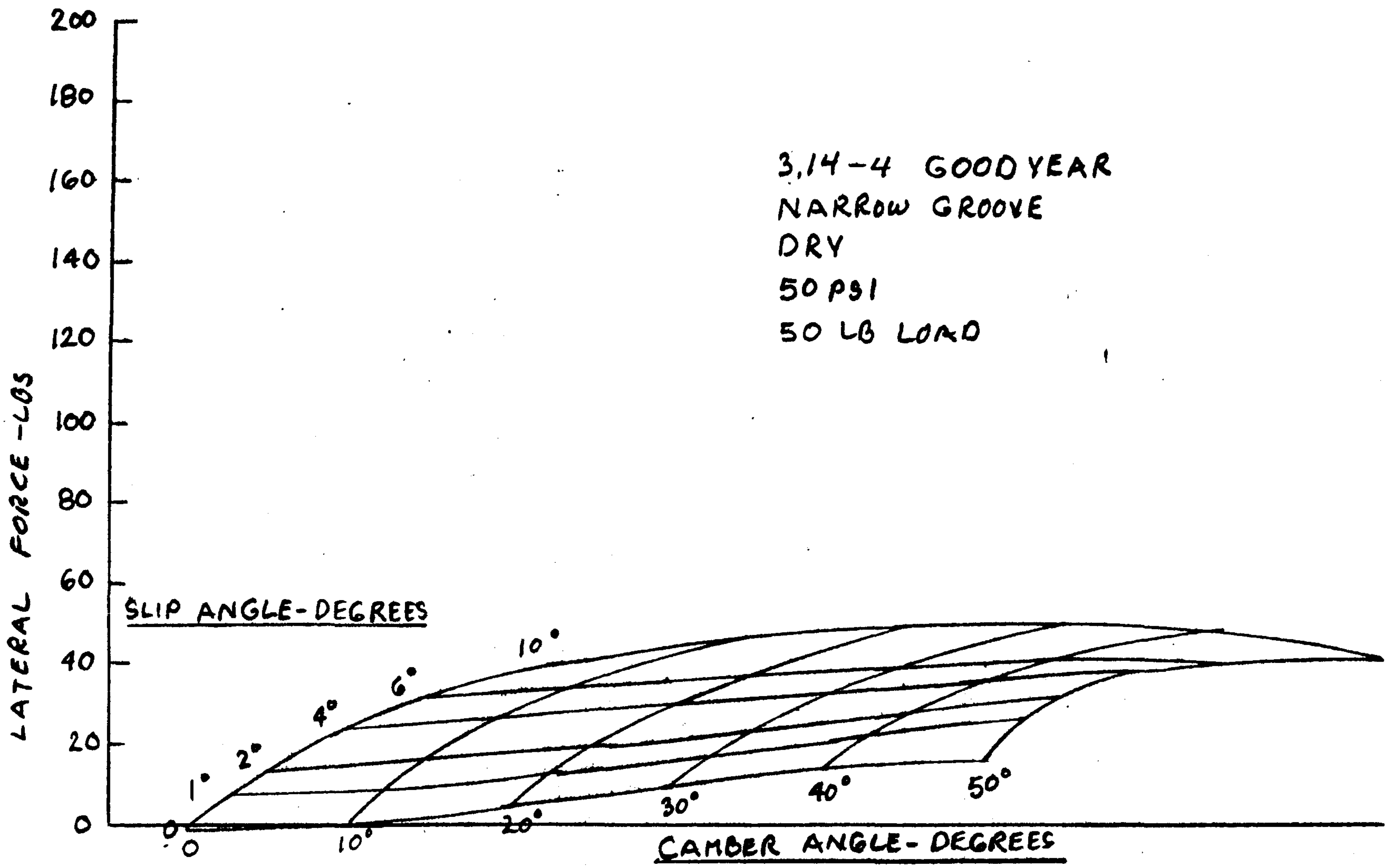
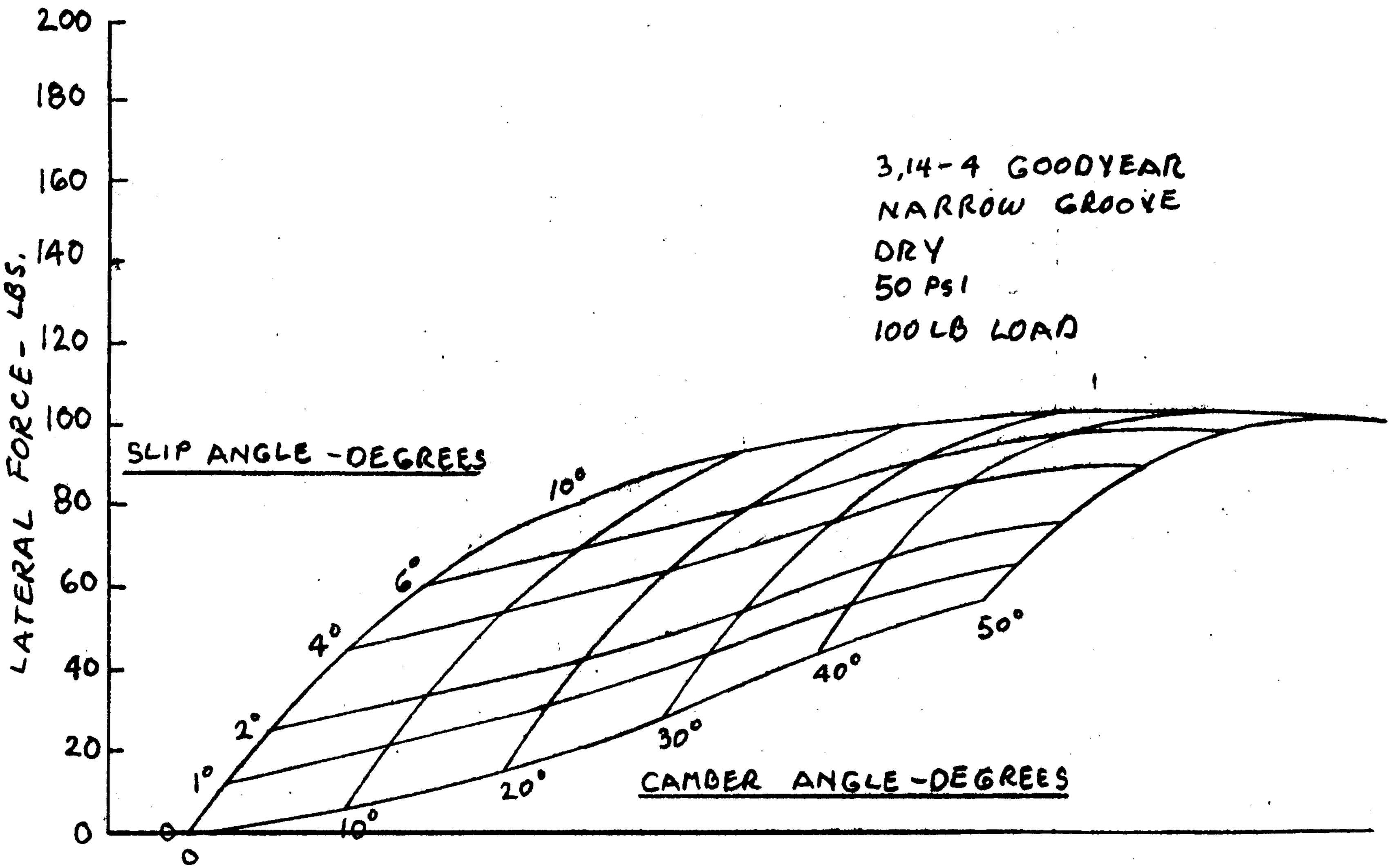

3.14-4 GOODYEAR
NARROW GROOVE
20 PSI
.010" WATER
20 MPH
100 LB. LOAD
LATERAL FORCE - LBS.
0
20
40
60
80
100
120
140
160
180
200
SLIP ANGLE - DEGREES
0
1°
2°
4°
6°
10°
CAMBER ANGLE - DEGREES
10°
20°
30°
40°
50°


3.14-4 GOODYEAR
NARROW GROOVE
20 PSI
.010" WATER
20 MPH
150 LB. LOAD
SLIP ANGLE - DEGREES
LATERAL FORCE - LBS
CAMBER ANGLE - DEGREES
200
180
160
140
120
100
80
60
40
20
0
0
1°
2°
4°
6°
10°
0
10°
20°
30°
40°
50°



3.14-4 GOODYEAR
NARROW GROOVE
DRY
50 PSI
50 LB LOAD
LATERAL FORCE -LBS
SLIP ANGLE-DEGREES
CAMBER ANGLE-DEGREES
0
20
40
60
80
100
120
140
160
180
200
0°
1°
2°
4°
6°
10°
0
10°
20°
30°
40°
50°

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| TITLE | DATE | CHK. | REVISIONS—WAS | LET. | DATE |
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| | BY | | | | |
| REF. DWG. NO. | PROJECT | | | | |
| REQUESTED BY | DEPT. | | | | |

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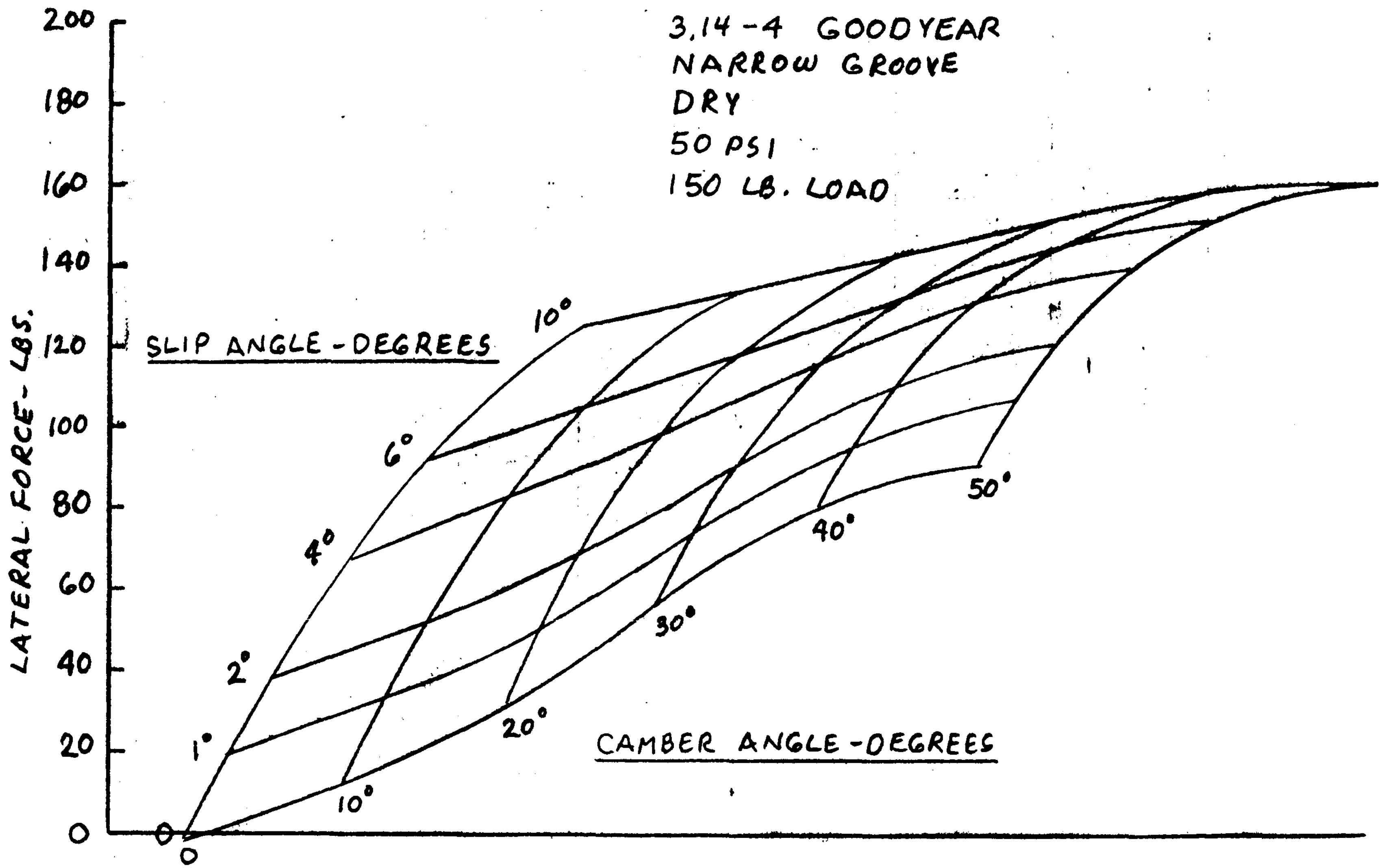

3.14-4 GOODYEAR
NARROW GROOVE
DRY
50 PSI
150 LB. LOAD
LATERAL FORCE - LBS.
SLIP ANGLE - DEGREES
CAMBER ANGLE - DEGREES
0
20
40
60
80
100
120
140
160
180
200
0
1°
2°
4°
6°
10°
10°
20°
30°
40°
50°

RER
4-23-75

# SUMMARY

# STEADY STATE STEER TORQUE ANALYSIS

# PROJECT 300

AN EQUATION FOR BICYCLE STEER TORQUE, PRESENTED IN CALSPAN REPORT ZN-5431-V-1 BY R.S. RICE, WAS SIMPLIFIED AND APPLIED TO SOME ESTIMATED PARAMETERS FOR THE PROJECT 300 VEHICLE. CONCLUSIONS ARE THE FOLLOWING:

1. GYROSCOPIC EFFECTS ARE NEGLIGIBLE FOR THE PROJECT 300 TIRE AND WHEEL SIZE.
2. STEER TORQUE FROM THE CAMBER-FORK GEOMETRY INTERACTION IS SMALL COMPARED TO STEER TORQUE FROM THE STEER ANGLE-FORK GEOMETRY INTERACTION.
3. CALCULATED STEER TORQUE GRADIENT RESULTS FROM A "SMALL DIFFERENCE OF LARGE NUMBERS" EQUATION. A VALID RESULT WILL REQUIRE ACCURATE VEHICLE PARAMETERS
4. TORQUE DUE TO TIRE LATERAL FORCE AND TRAIL IS THE LARGEST TERM ALTHOUGH TORQUE DUE TO STEER ANGLE-FORK GEOMETRY IS NEARLY AS LARGE, (DEPENDING ON THE VALIDITY OF THE ASSUMED PARAMETERS)

(NOTE: THE RICE EQUATION WAS NOT REDERIVED OR CHECKED. SIGN CONVENTIONS ARE NOT VERY CLEAR IN ZN-5431-V-1.)

A SUMMARY OF THE EQUATIONS IS THE FOLLOWING: (SEE ATTACHED TEXT FOR SYMBOL DEFINITIONS)

STATIONARY (PARKED) STEER TORQUE

$$T_p = \left[Z_F t - W_S f\right]\left[\phi + \sin\sigma\,\delta\right]$$

DYNAMIC STEER TORQUE GRADIENT - (LESS SMALL TERMS)

$$\frac{\partial T}{\partial A_Y} = \left[\frac{Mbt}{\ell} + M_S f\right] G + \left[Z_F t - W_S f\right]\left[\frac{1}{1+A_Y^2} + \left(57.3\frac{\ell G}{V^2} + K\right)\sin\sigma\right]$$

4-21-75

# STEADY STATE STEERING TORQUE ANALYSIS

CALSPAN CORPORATION HAS PROVIDED AN EQUATION FOR THE STEADY STATE STEERING TORQUE REQUIRED BY A CAMBER STABILIZED VEHICLE WITH A FORK RAKE ANGLE AND OFFSET, OPERATING IN A TURN. THE EQUATION IS THE FOLLOWING:

$$(1)\quad T = t\left[C_{\alpha F}\left(\delta \cos\sigma - \beta - \frac{ar}{V}\right) + C_{\phi F}(\phi + \delta \sin\sigma)\right] + M_S f V r + \frac{i_F}{R} V r \sin\sigma + \delta(Z_F t - W_S f)\sin\sigma + \phi(Z_F t - W_S f)$$

WHERE:

$T$ = STEERING TORQUE - FT.LB.

$V$ = FORWARD VELOCITY - FT/SEC

$C_{\alpha F}, C_{\phi F}$ = CORNERING AND CAMBER STIFFNESSES - LB/RAD

$\delta$ = STEER ANGLE - RAD

$\sigma$ = STEER AXIS INCLINATION - RAD.

$\beta$ = SIDESLIP ANGLE - RAD.

$a$ = DISTANCE FROM FRONT WHEEL C.L. TO C.G.

$r$ = YAW RATE - RAD/SEC

$\phi$ = CAMBER ANGLE - RAD.

$M_S$ = STEERING MASS - SLUGS, $W_S$ = STEERING WEIGHT - LB

$f$ = FORK MASS OFFSET - FT

$i_F$ = FRONT WHEEL SPIN INERTIA - LB-FT-SEC$^2$

$R$ = TIRE ROLLING RADIUS - FT.

$Z_F$ = GROUND REACTION FORCE - LBS. $\left\{\begin{array}{l}\text{DEFINED BY} \\ \text{CALSPAN AS A} \\ \text{NEGATIVE QUANTITY}\end{array}\right.$

$t$ = MECHANICAL TRAIL - FT.

IT IS IMPORTANT TO NOTE THE PHYSICAL SIGNIFICANCE OF EACH TERM IN THE EQUATION AS DERIVED BY CALSPAN. FROM THIS IT WILL BE CLEAR HOW THIS RELATIONSHIP CAN BE SIMPLIFIED AND QUANTITATIVELY INTERPRETED.

1. $t\left[C_{\alpha F}\left(\delta \cos\sigma - \beta - \frac{ar}{V}\right) + C_{\phi f}\left(\phi + \delta \sin\sigma\right)\right]$ - TIRE LATERAL FORCE MULTIPLIED BY MECHANICAL TRAIL.

2. $M_s f V r$ - A MOMENT DUE TO CENTRIFUGAL FORCE ASSOCIATED WITH TURNING. THIS ACTS THROUGH THE OFFSET OF THE FORK SYSTEM CENTER OF GRAVITY.

3. $\frac{i_F}{R} V r \sin\sigma$ - A GYROSCOPIC TERM ASSOCIATED WITH STEADY TURNING AND WHEEL SPIN VELOCITY.

4. $\delta(Z_F t - w_s f)\sin\sigma$ - A STATIC UNBALANCE TERM RESULTING FROM STEER ANGLE, FORK GEOMETRY, AND CG OFFSET.

5. $\phi(Z_F t - w_s f)$ - A STATIC UNBALANCE TERM RESULTING FROM CAMBER ANGLE, FORK GEOMETRY, AND CG OFFSET.

EQUATION (1) IS WRITTEN IN TERMS OF THE CONVENTIONAL VEHICLE MOTION VARIABLES OF YAW AND SIDESLIP. AS SUCH, IT IS RELATIVELY DIFFICULT TO EVALUATE AND INTERPRET. IT HAS, IN THE PAST BEEN MORE CONVENIENT TO

EXPRESS A RELATIONSHIP OF THIS NATURE IN TERMS OF LATERAL ACCELERATION.

FOR STEADY STATE TURNING, FRONT TIRE LATERAL FORCE IS RELATED TO THE PORTION OF WEIGHT ON THE FRONT WHEEL AND LATERAL ACCELERATION $\frac{Wb}{\ell} A_y$. IT IS NOT NECESSARY TO RELATE THIS TO SLIP, CAMBER, YAW RATE, OR STEER ANGLE. THE TERM $Vr$ IS LATERAL ACCELERATION IN FT/SEC², SO $\frac{Vr}{G} = A_y$ EXPRESSED IN G'S. FOR EQUILIBRIUM, $\phi = \tan^{-1} A_y$, MAKING THESE SUBSTITUTIONS REDUCES EQUATION #1 TO:

$$(2)\quad T = \frac{Wbt}{\ell} A_y + M_s f G A_y + \frac{i_F}{R} \sin\sigma\, G A_y$$
$$+ (Z_F t - W_s f) \sin\sigma\, \delta + (Z_F t - W_s f) \tan^{-1} A_y$$

FURTHER REARRANGEMENT GIVES:

$$(3)\quad T = \left[\frac{Mbt}{\ell} + M_s f + \frac{i_F}{R} \sin\sigma\right] G A_y$$
$$+ \left[Z_F t - W_s f\right]\left[\tan^{-1} A_y + \sin\sigma\, \delta\right]$$

EQUATION (3) CAN BE USED IN ITS PRESENT FORM. NOTE THAT IF THE VEHICLE IS NOT MOVING,

(4) $T = \left[ Z_F t - w_s f \right] \sin\sigma \, \delta$

WHICH EXPRESSES THE STATIONARY STEER UNBALANCE PHENOMENA. FOR THIS TYPE OF ANALYSIS, IT WOULD BE BETTER TO CONVERT BACK TO:

(5) $T = \left[ Z_F t - w_s f \right] \left[ \phi + \sin\sigma \, \delta \right]$

TO EXPRESS THE EFFECT OF CAMBER FOR THE PARKED CONDITION.

FOR THE DYNAMIC CASE, WE NEED A STEER TORQUE GAIN EQUATION OBTAINED FROM THE DERIVATIVE OF EQUATION (3)

(6) $$\frac{\partial T}{\partial A_Y} = \left[ \frac{M b t}{\ell} + M_s f + \frac{i_F}{R} \sin\sigma \right] G + \left[ Z_F t - w_s f \right] \left[ \frac{1}{1 + A_Y^2} + \sin\sigma \frac{\partial \delta}{\partial A_Y} \right]$$

THE TERM $\frac{\partial \delta}{\partial A_Y}$ HAS BEEN DERIVED IN EARLIER TIRE ANALYSIS. THIS IS THE WELL KNOWN GAIN EQUATION;

(7) $\frac{\partial \delta}{\partial A_Y} = 57.3 \frac{\ell G}{V^2} + K$

WHERE K IS UNDERSTEER °/G.

SUBSTITUTION GIVES:

(8) $$\frac{\partial T}{\partial A_Y} = \left[\frac{M b t}{\ell} + M_S f + \frac{i_E}{R} \sin\sigma\right] G + \left[2_F t - w_S f\right]\left[\frac{1}{1+A_Y^2} + \sin\sigma\left(57.3\frac{\ell G}{V^2} + K\right)\right]$$

ON THE SURFACE THIS MAY NOT APPEAR TO BE A SIGNIFICANT SIMPLIFICATION OVER EQUATION (1) BUT NOTE THAT THE INDEPENDENT VARIABLES HAVE BEEN REDUCED TO SPEED AND LATERAL ACCELERATION IN G'S. THE OTHER TERMS ARE VEHICLE DESIGN PARAMETERS. FURTHER SIMPLIFICATION DEPENDS ON THE MAGNITUDES OF THESE VARIOUS TERMS.

ASSUMED PARAMETERS

$M = \frac{250 \text{ LB}}{32.2} = 7.8 \text{ SLUGS}$

$b = \frac{5.5}{12} = .46 \text{ FT}$

$l = 3 \text{ FT}$

$t = 5/12 = .42 \text{ FT}$

$W_S = 32 \text{ LB}, \quad M_S = 1 \text{ SLUG}$

$f = .2 \text{ FT}$

$i_F = \frac{3.4}{32.2} \times \left(\frac{2}{12}\right)^2 = .0029 \text{ LB-FT-SEC}^2$

$\sigma = 25°, \sin\sigma = .42$

$Z_F = -38 \text{ LB.}$

$R = 4.3/12 = .36 \qquad K = 2$

THESE PARAMETERS SHOULD BE VIEWED AS APPROXIMATIONS.

SUBSTITUTION INTO EQUATION (8) GIVES THE FOLLOWING:

$$(9) \quad \frac{dT}{dA_Y} = 32.2\left[\frac{7.8(.46)(.42)}{3} + 1(.2) + \frac{.0029(.42)}{.36}\right] + \left[-38(.42) - 32(.2)\right]\left[\frac{1}{1+A_Y^2} + .42\left(\frac{57.3(3)32.2}{V^2} + 2\right)\right]$$

COMBINING GIVES.

$$(10)\quad \frac{\partial T}{\partial A_Y} = 32.2\left[.5 + .2 + .0034\right] + \left[16 - 6.4\right]\left[\frac{1}{HA_Y^2} + .42\left(\frac{5520}{V^2} + 2\right)\right]$$

FOR $A_Y = .5$, $V = 20$ MPH

$$(11)\quad \frac{\partial T}{\partial A_Y} = \left[16.1 + 6.4 + .11\right] - 22.4\left[.67 + .42(.146)\right]$$

$$= \left[16.1 + 6.4 + .11\right] - 22.4\left[.67 + .061\right]$$

$$= \left[16.1 + 6.4 + .11\right] - \left[15.0 + 1.37\right]$$

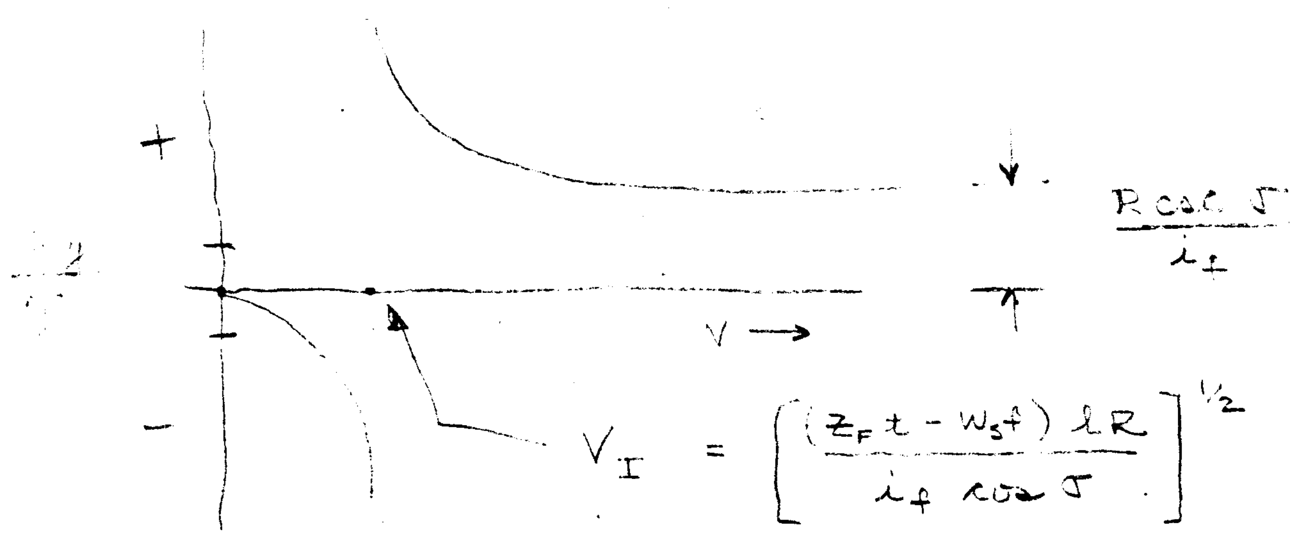
$$= 22.6 - 16.4$$

$$= 6.2\ \frac{\text{FT-LB}}{\text{G}}$$

FROM THIS, WE MIGHT CONCLUDE THE FOLLOWING FOR PARAMETERS APPLICABLE TO THE PROJECT 300 VEHICLE.

1. THE GYROSCOPIC TERM IS NEGLIGIBLE FOR THE SMALL TIRE AND WHEEL.
2. THE CAMBER-FORK GEOMETRY TERM IS SMALL COMPARED TO THE STEER-FORK GEOMETRY TERM.
3. OTHER TERMS ARE OF SUFFICIENT SIZE TO BE INCLUDED.
4. NET STEER TORQUE GRADIENT IS THE RESULT OF THE DIFFERENCE BETWEEN THE TRAIL EFFECT AND FORK GEOMETRY EFFECT - EACH OF WHICH IS LARGE. THE CALCULATED TORQUE GRADIENT WILL BE DEPENDENT ON THE ACCURACY OF THE VEHICLE PARAMETERS.

1




$$V_I = \left[\frac{(Z_F t - W_s f)\, l R}{i_f \cos\sigma}\right]^{1/2}$$

* Also consider q/T and E/T expressions

what is the function of $V_I$ with respect to operating [illegible]

1. y intercept = $\dfrac{(Z_F t - W_s f)\, l \tan}{IR}$

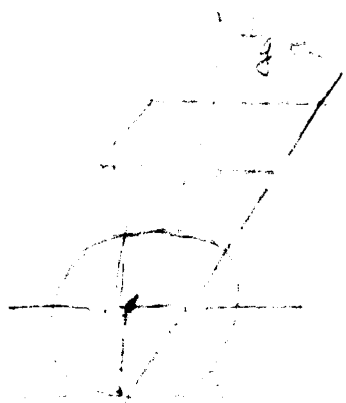
x intercept = $1/K_T IR$ = $\dfrac{(Z_F t - W_s f)}{i_f \cos\sigma\, IR}$

[illegible]

[illegible]

[illegible] be constant with [illegible]

$f_g =$ [illegible]



[illegible]