general.

His technique for deriving the equations of motion was to first linearize the equations of motion of a rolling hoop and then “add on” the trailer effects due to the remaining parts of the bicycle using fairly casual arguments. While his brief verbal justifications sound valid, in fact almost no terms in the equations are exactly correct. We did not make the effort to trace his errors, but note that there may have been a major mistake in the kinematical treatment (which is not spelled out very explicitly): the headings γ and θ of the rear and front assemblies are defined relative to the track line, but then they appear to be treated as coordinates relative to inertial space in the equations.

We compared his equation (4) to our steer equation and his equation (5) to our lean equation, and found that his equations differ significantly in almost every term when compared to those presented in Chapter III. Kis equations would also disagree with Bower’s if simplified for Bower’s model.

Pearsall does not say if he compared his equations to Bower’s, and he does not refer to any other works.

Timoshenko and Young, 1948

In this textbook on advanced dynamics, Timoshenko and Young derived a nonlinear (large-angle) lean equation for a simplified Basic bicycle model having only a point mass in the rear part of the bicycle, and a steer angle controlled by the rider. Their model neglects wheel inertias, steering axis tilt, trail and front-mass offset from the steering axis. When linearized, we find this lean equation agrees with our lean equation simplified for an equivalent configuration.

Döhring, 1955

In 1955, in order to more generally analyze the stability of motorcycles and motorscooters, Döhring extended Sommerfeld and Klein's (S & K) [1903] linearized equations for the Basic bicycle model by allowing the mass distribution of the front assembly to be fully general. Just as S & K did, Döhring used Newton's Laws to derive the equations of motion in linearized form, rather than linearizing from nonlinear equations as Whipple had.

Dohring's final equations were found to be in exact agreement with those derived in Chapter 111. In order to compare his equations to ours we made the following substitutions in his equations (29) and (30) of his [1955] paper,

$$\psi = \gamma \cos \sigma$$

$$\theta_1 = \theta_2 - \gamma \sin \sigma$$

where γ is steer angle (our ψ) and θ_2 is lean angle (our χ_r). When these substitutions are made Döhring's equation (30) is exactly our lean equation. Our steer equation results from the linear combination of Dohring's equation (31) and (30). Using Döhring's notation this combination is as follows:

$$\frac{(eq.\,31)}{1} + \frac{c_1 \sin\sigma (eq.\,\mathbf{30})}{1} = -M_d = our\ \ M_\psi$$

Although not rigorous in how his linearizations are made, Dohring's derivation was fairly easy to follow, and offers a good physical description of the variables and