needed to derive the correct potential energy.) By eliminating the effects of viscous damping in the steering column, and making the above corrections N & F's final equations of motion can be brought into agreement with those derived in Chapter III.

N & F refer to Döhring[4] and state that their equations agree in form, but it is unlikely they meant term for term as we have found them to be in disagreement. They also refer to a Russian book by Loǐcjanskiǐ and Lurk [1935] when analyzing a simplified model of an uncontrolled bicycle on **p. 355.** Because this reference was not available, it is not known if agreement actually exists, however it seems probable because N & F's equations become correct when simplified in this way. N & F do not mention any other bicycle-related works, although their massive reference list includes Carvallo [1901].

Singh and Goel, 1971

In January 1971 Singh and Goel (S & G) add steer damping to the Basic bicycle model in analyzing a Rajdoot motor scooter. In their analysis they claim to use Döhring's [1955] linearized equations of motion (which we have found to be correct) with a steering torque proportional to the the time derivative of the steer angle (viscous damping). We have not rigorously compared term by term but casual observation shows that the equations are in the same format as those of Dohring [1955].

[4] See **p.** 361 of their text.

S & **G** refer to Pearsall [1922], Timoshenko [1948], Döhring [1955], Collins [1963], and Singh [1964], but make **no** comparison to their equations of motion.

Sharp, ***1971***

In August 1971 Sharp, who apparently began working on the equations of motion while at the B. S. **A.** motorcycle company, published a paper presenting his version of the linearized equations of motions for the motorcycle. In his Lagrangian approach rather than using the method presented by Neĭmark and Fufaev in Chapter III, he explicitly allows the vertical force from the ground on the front wheel (Z_f) to do work on the bicycle. **For** this reason Z_f appears in his expressions for the generalized forces. In this way he accounts for the change in potential energy of the bicycle when steered. The nonlinear equations he presents are actually only approximations for this reason.

Allowing for wheel side slip, and incorporating the work done by the vertical force on the front wheel, he derived Lagrange's equations with generalized forces at the wheels' contact with the ground. These resulted in four equations of motion, incorporating front and rear tire side forces, **which** govern lateral motion, yaw, **roll,** and steer of the motorcycle. They which appear in his paper starting'at the bottom of **p.** 327 (no equation numbers are **given).** These equations are correct **as** far **as** we know.

However, when assuming that the tires have infinite stiffness (no side slip), which reduces the number of equations from four to two, an algebraic mistake and