S & G refer to Pearsall [1922], Timoshenko [1948], Döhring [1955], Collins [1963], and Singh [1964], but make no comparison to their equations of motion.

Sharp, ***1971***

In August 1971 Sharp, who apparently began working on the equations of motion while at the B. S. A. motorcycle company, published a paper presenting his version of the linearized equations of motions for the motorcycle. In his Lagrangian approach rather than using the method presented by Neĭmark and Fufaev in Chapter III, he explicitly allows the vertical force from the ground on the front wheel (Z_f) to do work on the bicycle. For this reason Z_f appears in his expressions for the generalized forces. In this way he accounts for the change in potential energy of the bicycle when steered. The nonlinear equations he presents are actually only approximations for this reason.

Allowing for wheel side slip, and incorporating the work done by the vertical force on the front wheel, he derived Lagrange's equations with generalized forces at the wheels' contact with the ground. These resulted in four equations of motion, incorporating front and rear tire side forces, which govern lateral motion, yaw, roll, and steer of the motorcycle. They which appear in his paper starting'at the bottom of p. 327 (no equation numbers are given). These equations are correct as far as we know.

However, when assuming that the tires have infinite stiffness (no side slip), which reduces the number of equations from four to two, an algebraic mistake and several typographical errors occur in the Appendix 11. As a result the steer equation (the second equation) is incorrect. The algebraic error made by Sharp results in the incorrect cancellation of the following term (in his notation),

$$2[M_f ek + I_{fz}\cos\epsilon + M_f eb]l_1 t\,\ddot{\delta}$$

We also make note of the following typos: the x_1^2 in the lean equation of Appendix II should read $\dot{x}_1^2$; there is an extra parathesis in the ninth term of the fourth equation in Appendix 1 section entitled "Linear equations of motion"; the term $\dot{x}_1 \cos\epsilon\ddot{\delta}$ in the expression for $\ddot{\psi}$ in Appendix II should read $\dot{x}_1 \cos\epsilon\dot{\delta}$; I_{fy} should read $i_f y$ in the $\dot{\phi}$ term of the steer equation of appendix 2; and finally terms involving $\frac{i_{fy}}{R_f} l_1 t\dot{x}_1 \sin\epsilon$ in the $\dot{\delta}$ term of the steer equation can be eliminated as they cancel one another.

Sharp also makes the slightly restrictive assumption that one principal axis of the center of mass moment of inertia tensor of the front assembly is parallel to the steering axis. Thus the equations in his paper, when corrected, are a subset of those derived in Chapter 111. Sharp refers to the work of Whipple [1899], Pearsall [1922] and Collins [1963], but does not compare his equations to theirs.

Roland, 1971

In 1971 Roland published a report written for the Schwinn Bicycle Company containing a extensive nonlinear computer simulation study.[5] In this report Roland derived nonlinear equations that represent the motion of a bicycle with tire side slip

[5] This report was based on work performed for a National Commission on Product Safety research contract. See Roland [1970].