ON THE DYNAMICAL PRINCIPLES OF THE MOTION OF VELOCIPEDES.

By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.S.

1. *Division of the subject.*—The purpose of this communication is to give an elementary explanation of the dynamical principles upon which depends the motion of velocipedes (commonly called "bicycles") that run on two wheels, placed one before the other. In regulating the motion of those vehicles three objects are to be accomplished: balancing, steering, and propulsion. Balancing consists in keeping the wheels in such a position as to support the load when moving in a straight line, or in a curve of a given radius; steering, in changing the path in which the vehicle with its load moves, from a straight line to a curve of a given radius, and swerving in a given direction, or from a curve to a straight line, or from one curve to another curve of a different radius; and propulsion, in applying the power required in order to overcome the resistance opposed to the motion, and so to keep up the speed.

SECTION I.—BALANCING.

2. *General explanations.*—From the general principle in dynamics, that the motion of the common centre of mass of a set of bodies cannot be altered by the mutual actions of those bodies, it follows that the rider of the velocipede has no power, by any force he may exert on the parts of the machine, or by any attitude he may assume, of directly affecting the motion of the common centre of mass of himself and the velocipede: a point which, in what follows, will be called simply the *centre of mass*. The only forces which can and do affect that motion are forces exerted by external bodies upon the velocipede and its load: viz., the force of gravity, exerted by the whole earth, which may be treated as if its action were concentrated at the centre of mass: the pressure and friction exerted by the roadway on the rims of the wheels; and the pressure and resistance of the air. The effect of this last class of forces will be considered separately further on. The control possessed by the rider over the movements of the centre of mass is indirect, and arises from the power which he has of varying at will the positions and directions of the forces exerted by the roadway on the rims of the wheels.

3. *Transverse friction.*—The forces which directly take effect in balancing the machine are the transverse components of the friction exerted by the roadway on the wheels; in other words, the forces with which the roadway resists any tendency of the wheels to slip sideways. The greatest possible amount of the transverse friction bears a definite ratio to the load, depending on the roughness of the roadway and on the nature of its material and of that of the wheels. That ratio, or *coefficient of transverse friction*, is the tangent of a certain angle, well-known by the name of the *angle of repose*. That angle expresses the greatest possible obliquity of the pressure exerted between the wheel rims and the roadway; and it also expresses the greatest angle at which the wheels, when running on a roadway that is level across, can lean over sideways without slipping, and so making it impossible to balance the velocipede. What the angle of repose is for the wheels of actual velocipedes can be ascertained with precision by experiment only. It probably does not differ greatly from the angle of repose of iron upon stone, which was found in Morin's experiments to range from 17 deg. to 35 deg., and its tangent from 0·3 to 0·7. Throughout this communication it will be assumed that the obliquity of the pressure between the wheel rims and the roadway does not exceed the angle of repose; in supposing the rider to balance it correctly. The radii A C and B C are obviously the projections on the plane of the roadway of the axes of rotation of the two wheels.

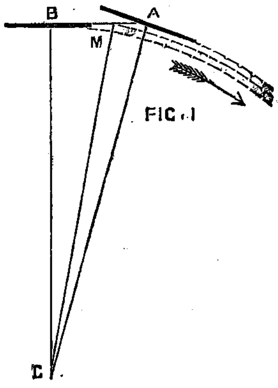


FIG. 1

6. *The base point, and its track.*—Let M, in Fig. 1, be the foot of a perpendicular let fall from the centre of mass upon the wheel base A B. This may be called, for brevity's sake, the *base point;* and it is the point traversed by the resultant of the two upward pressures exerted by the roadway on the wheels at the points of support A and B. When the wheel planes coincide the track of the base point is obviously a straight line coinciding with the two wheel tracks. When the wheel planes make an angle with each other the track of the base point is a circle concentric with the two wheel tracks, being described about the common centre C, Fig. 1.

7. *Keeping the balance* of a velocipede consists in so guiding the wheels that the base point shall either always be in the line of action of the load—that is, of the resultant of gravity and centrifugal force, which traverses the centre of mass—or shall have its deviations from that line of action quickly corrected. It is effected through the power which the rider has, by means of the steering-bar, of placing the fore-wheel plane at any required angle of inclination to the hind-wheel plane, and thus of giving any required curvature, towards the right or towards the left, to the track of the base point. When the track of the centre of mass is a straight line the track of the base point ought to be a straight line in the same vertical plane. When the track of the centre of mass is a circle the track of the base point ought to be a parallel circle, whose radius is a little greater than that of the track of the centre of mass. The difference between those radii will be considered in the next article. It is impossible for any rider absolutely to prevent the base point from deviating from its proper track; but a practised rider corrects any deviation which may occur with a promptitude depending on his skill. This he does by making the fore wheel plane deviate in a direction contrary to that of the deviation of the base point which is to be corrected. The time required in order to correct a given deviation of the base point from the line of action of the load, by means of a given angular deviation of the steering-bar and fore wheel plane, is inversely proportional to the speed with which the velocipede is running. The angular deviation of the fore wheel plane required in order to correct a given deviation of the base point in a given time varies inversely as the square of the speed. Hence, the well-known facts, that the higher the speed the easier it is to keep the balance of the velocipede, and that at very low speeds the keeping of the balance becomes difficult, or even impossible. The art of managing the steering-bar so as to guide the base point in its proper track can, of course, be learned by practice alone; but a knowledge of the principles on which that art depends may still be useful to the learner, by guarding him against erroneous practices, and by enabling him to know the causes of the success or failure of his early attempts at balancing the vehicle. Practised and dexterous riders are able to balance the velocipede without the use of the steering-bar: the relative positions of the wheels and consequent form of the

on the horizontal ordinate when the same rate of expansion is maintained. The engines are of the common direct-acting steam hammer class, having two 72in. cylinders without steam jackets, and 4ft. 6in. stroke. A small surface condenser has been used in preference to a larger one, as it was considered more economical to have an inferior vacuum and hot feed-water than to have a few inches more vacuum with cold feed-water, besides requiring large power to drive large circulating pumps. There are 3940 brass tubes of 7ft. length and $\frac{5}{8}$ inside ($\frac{3}{4}$in. bare outside) diameter, giving a total cooling surface of 5200 square feet, or 2·11 square feet per average indicated horse-power, which is equal to 1006 per cent. of the total grate, and 39 per cent. of the total heating surface of the boilers. The temperature of the feed-water averaged, for the sixty-three watches of the voyage, 134 deg. Fah. The "duty" of the engines, in the full meaning of that term, could not of course be ascertained for want of the actual horse-power, to be ascertained only by dynamometrical experiments. If the estimate of the duty from the indicated horse-power, which, of course shows only the utilisation of the steam by the distributing arrangements, is of any value, it may be easily determined from the quantity of heat consumed by the engine in the form of steam. It will be shown, when treating of the performance of the boilers, that the amount of heat produced by the boilers from each pound of coal consumed was 9643 calories, which, in a perfect engine, would be converted into 9643 × 772 foot-pounds of work. Now, taking the indicated horse-power, in want of the actual or dynamometrical horse-power, we get, with a consumption of 2·92 lb. of coal per indicated horse-power per hour, $\frac{33000 \times 60}{2 \cdot 92}$ foot-pounds of work for every pound of coal consumed. This comes up to 92 per cent. of the work due to the heat supplied by the engine for the same amount of coal.

Performance and "Duty" of Boilers.—There are four multitubular fire-tube boilers, two on each side of the ship, with the stokehole placed between them. They are each 24ft. long longitudinally, by 10ft. 7in. deep, and have altogether, twenty-four furnaces, with an aggregate grate surface of 516 square feet, or 0·206 square feet per average indicated horse-power. There are 13,200 square-feet of aggregate heating surface, or 5·2 square feet of heating surface per average indicated horse-power. The superheating apparatus consists of five larger tubes in the uptake, with an aggregate surface of 560 square feet exposed to the action of the hot gases, and this amount, at the rate of 0·22 square feet per indicated horse-power, or 0·042 per cent. of the total heating surface, superheats the steam to an extent sufficient to keep its temperature, when the full working pressure is used, at about 300 deg. Fah. on entering the cylinders. The "duty" of a boiler is to take all the heat out of the fuel and deliver it to the engines without allowing any losses to take place by blowing off, radiation, &c. The fuel used in the Westphalia was a fair quality of Welsh coal, containing, say, 85·5 per cent. of carbon, and capable of evaporating, in an ideal boiler which would consume all the smoke and radiate no heat whatever, about 14 lb. of water at the boiling point of 212 deg. Fah. The latent heat of steam at this temperature, in its saturated state, is 966 calories; therefore, the amount of heat obtainable from one pound of coal is 966 × 14 = 13,524 calories, and the "duty" of a boiler would be the percentage of this amount which it realises from one pound of coal. In the Westphalia's boilers the duty may be ascertained as follows:—The steam pressure, on the verse components of the friction exerted by the roadway on the wheels; in other words, the forces with which the roadway resists any tendency of the wheels to slip sideways. The greatest possible amount of the transverse friction bears a definite ratio to the load, depending on the roughness of the roadway and on the nature of its material and of that of the wheels. That ratio, or *coefficient of transverse friction*, is the tangent of a certain angle, well-known by the name of the *angle of repose*. That angle expresses the greatest possible obliquity of the pressure exerted between the wheel rims and the roadway; and it also expresses the greatest angle at which the wheels, when running on a roadway that is level across, can lean over sideways without slipping, and so making it impossible to balance the velocipede. What the angle of repose is for the wheels of actual velocipedes can be ascertained with precision by experiment only. It probably does not differ greatly from the angle of repose of iron upon stone, which was found in Morin's experiments to range from 17 deg. to 35 deg., and its tangent from 0·3 to 0·7. Throughout this communication it will be assumed that the obliquity of the pressure between the wheel rims and the roadway does not exceed the angle of repose; in other words, that the wheel rims do not slip sideways.

4. *Axes and middle planes of the wheels.*—In a two-wheeled velocipede of ordinary construction the axis of the hind wheel is fixed relatively to the frame of the vehicle. The centre of the fore wheel is also fixed relatively to the frame, being situated in the middle plane of the hind wheel. The axis of the fore wheel is adjustable by means of the steering-bar, so as either to be parallel to that of the hind wheel, or to make any required angle with it to the right or to the left. When the axes are parallel the middle planes of the two wheels coincide; when the axes make an angle the middle planes of the wheels make an equal angle in the same direction. Those planes may be called the *hind wheel plane* and the *fore wheel plane* respectively. The hind wheel plane contains the two *points of support:* that is, the middle points of contact of the rims of the two wheels with the roadway; it also contains the centre of mass when the load is symmetrically distributed. The *wheel base* is the straight line joining the points of support; it lies in the intersection of the hind wheel plane with the plane of the roadway.

5. *Figures of the wheel tracks.*—The figures of the wheel tracks, or lines traced by the rims of the wheels as they run on the roadway, depend on the angle which two tangents to the wheels in the plane of the roadway make with each other. That angle is always nearly, and sometimes exactly, equal to the angle made with each other by the wheel planes. The tangent to the hind wheel coincides with the wheel base. When the wheel planes coincide with each other the two tangents lie in one straight line, which is the track of both wheels. When the tangents to the wheels in the plane of the roadway make an angle with each other the wheel tracks are a pair of concentric circles touching those tangents; and the common centre and the radii of those circles are found by the following construction:—In Fig. 1 let the plane of the figure represent the plane of the roadway; let A be the point of support of the fore wheel, B that of the hind wheel, and A B the wheel base. Let the thick lines through A and B respectively represent tangents to the wheels, in the plane of the roadway; in other words, the traces of the wheel planes. Then perpendicular to those tangents respectively draw A C and B C, cutting each other in C; this point will be the required common centre. The track of the fore wheel will be a circle described about C with the radius C A; the track of the hind wheel will be a circle described about C with the radius C B. A vertical straight line through C will be the axis about which the velocipede revolves or sweeps round,

This he does by making the fore wheel plane deviate in a direction contrary to that of the deviation of the base point which is to be corrected. The time required in order to correct a given deviation of the base point from the line of action of the load, by means of a given angular deviation of the steering-bar and fore wheel plane, is inversely proportional to the speed with which the velocipede is running. The angular deviation of the fore wheel plane required in order to correct a given deviation of the base point in a given time varies inversely as the square of the speed. Hence, the well-known facts, that the higher the speed the easier it is to keep the balance of the velocipede, and that at very low speeds the keeping of the balance becomes difficult, or even impossible. The art of managing the steering-bar so as to guide the base point in its proper track can, of course, be learned by practice alone; but a knowledge of the principles on which that art depends may still be useful to the learner, by guarding him against erroneous practices, and by enabling him to know the causes of the success or failure of his early attempts at balancing the vehicle. Practised and dexterous riders are able to balance the velocipede without the use of the steering-bar: the relative positions of the wheels, and consequent form of the wheel tracks, being regulated by the action of the feet on the cranks, or otherwise. The principles, however, of the art of balancing are the same in all cases.

(*To be continued.*)

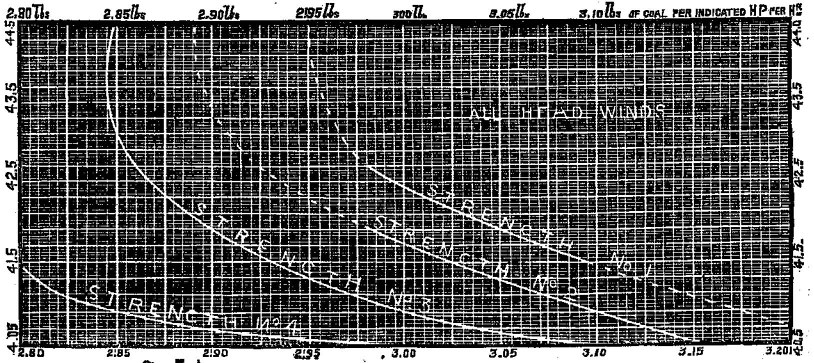
STEAMSHIP PERFORMANCE AND ECONOMY IN THE ATLANTIC MAIL SERVICE.

By Walter C. Bergius, C.E.

(*Continued from page* 63).

Utilisation of Weight in Ship.—The ship's hull when launched, with her cabin fittings and screw shaft, displaced 1800 tons of water, or 37·6 per cent. of the load displacement at 20·75ft. draught on even keel. This amount of material (which in a Transatlantic steamer of a proportion of length to breadth of nearly 8·5 : 1, may appear moderate) was arranged, however, to resist the heavy strains thrown upon the ship longitudinally during the rough passage, very satisfactorily, and no heating of screw shaft journals occurred in the tunnel during the voyage.

FIG . 3



Performance and Duty of Machinery.—The performance of the engines is given—as far as the indicated horse-power is a measure of the performance—in diagram No. 1 "performance of engines," obtained, as stated above, from a number of indicator diagrams taken with all the different degrees of expansion used during the voyage, viz., cutting off the steam at from 35 to 45 per cent. of the stroke. The steam distributing gear consists of a double ported slide-valve and a gridiron expansion valve working on a separate face behind it, and its effects are such that the maximum amount of efficiency is obtained from a given quantity of steam, when the port shuts at from 40·7 to 42 per cent. of the stroke; and the minimum effect when the steam is cut off at about 40 per cent., as will appear from the diagram. The curve given applies to a pressure of steam + vacuum of 40 lb.; for a higher pressure the coefficients of the indicator diagrams would be slightly superior to those given

superheating apparatus consists of five larger tubes in the uptake, with an aggregate surface of 560 square feet exposed to the action of the hot gases, and this amount, at the rate of 0·22 square feet per indicated horse-power, or 0·042 per cent. of the total heating surface, superheats the steam to an extent sufficient to keep its temperature, when the full working pressure is used, at about 300 deg. Fah. on entering the cylinders. The "duty" of a boiler is to take all the heat out of the fuel and deliver it to the engines without allowing any losses to take place by blowing off, radiation, &c. The fuel used in the Westphalia was a fair quality of Welsh coal, containing, say, 85·5 per cent. of carbon, and capable of evaporating, in an ideal boiler which would consume all the smoke and radiate no heat whatever, about 14 lb. of water at the boiling point of 212 deg. Fah. The latent heat of steam at this temperature, in its saturated state, is 966 calories; therefore, the amount of heat obtainable from one pound of coal is 966 × 14 = 13,524 calories, and the "duty" of a boiler would be the percentage of this amount which it realises from one pound of coal. In the Westphalia's boilers the duty may be ascertained as follows:—The steam pressure, on the average for the whole voyage, was 26·8 lb. above the atmosphere, or 41·5 lb. above zero. Saturated steam of this pressure has a temperature of 269·3 deg. Fah., and weighs 0·1029 lb. per cubic foot. If steam is superheated under constant pressure its volume expands in proportion to its temperature above the point of absolute zero of temperature. Taking the latter at 462·28 deg. Fah., the total temperature of the saturated steam would be

$$\begin{array}{r} 269{\cdot}3 \\ +462{\cdot}28 \\ \hline \end{array}$$

Total .. 731·58 deg. Fah.,

and of the superheated steam,

$$\begin{array}{r} 300{\cdot} \\ +462{\cdot}28 \\ \hline \end{array}$$

Total .. 762·28 deg. Fah.

Therefore the weight of a cubic foot of saturated steam under 41·5 lb. pressure at 300 deg. Fah. would be

$$\frac{0{\cdot}1029 \times 731{\cdot}58}{762{\cdot}28} = 0{\cdot}094 \text{ lb.}$$

During the whole voyage the consumption of coal was 804 tons for 250 hours 2 minutes, or 15,002 minutes, *i.e.*,

$$\frac{804 \times 2240}{15002} \text{ lb. per minute.}$$

During each minute of the voyage the boilers performed the work of filling each cylinder 2 × 53·4 times to the extent of 42·2 per cent. of its capacity, or both cylinders