ON THE DYNAMICAL PRINCIPLES OF THE MOTION OF VELOCIPEDES.

By W. J. MACQUORN RANKINE, C.E., LL.D., F.R.S.

(*Continued from page* 129.)

SECTION II.—STEERING.

13. *General Explanations.*—Steering consists in causing the track of the centre of mass to change at will from a straight line to a curve in either direction, or from a curve to a straight line, or from a curve to a different curve. As has already been stated under the head of balancing, the rider has no direct power of changing the track of the centre of mass; but he does so indirectly by changing the positions of the wheel-tracks and base-track, and thereby causing the pressure exerted by the roadway against the rims of the wheels to supply the force which produces the required change of track. The operation of changing the track of the centre of mass consists of two processes: by the first the base-point is shifted until it has the deflection necessary in order to produce the new curvature of the track of the centre of mass; by the second, the track of the base-point is adjusted so as to preserve that deflection, and thereby maintain the balance of the vehicle.

14. *Production and Variation of Curvature.*—Referring back to Fig. 2, let the plane of the paper represent a vertical plane, cutting the track of the centre of mass m at right angles. Let that track in the first place be a straight line; then the base-point is in the position P, vertically below the centre of mass; and P m is the trace of the hind-wheel plane, and is the line of action of the upward supporting pressure exerted by the roadway.

Next, suppose that by the guidance of the fore-wheel the base-point is shifted sideways to the position M, in a direction away from the intended centre of curvature. The trace of the hind wheel plane, and the line of action of the supporting pressure exerted by the roadway, is now M m. That pressure, being now oblique, may be resolved into a vertical and a transverse component. The vertical component is balanced by the equal and opposite force of gravity. The transverse component is unbalanced; it constitutes a *deviating* or *centripetal force*, acting horizontally on the vehicle, in the direction $m\,c$; and it causes the centre of mass to describe a curve such that the centrifugal force is to gravity as P M is to m P. The radius of that curve may be calculated by the following formula (which is equation 1 in another shape)

$$r = m\,c = \frac{v^2 \cdot m\,\mathrm{P}}{g \cdot \mathrm{P\,M}} = \frac{v^2}{g \cdot \tan. \theta}; \quad . \quad . \quad . \quad (12.)$$

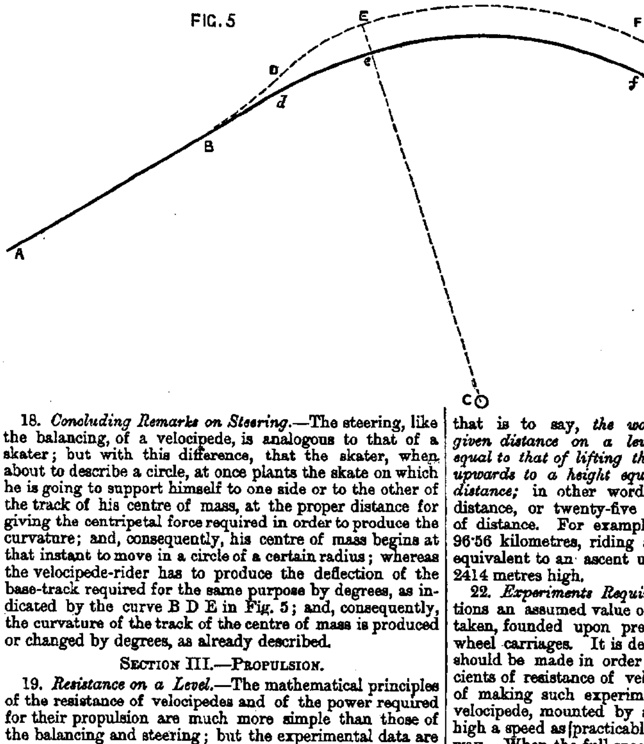
(where θ, as before, is the angle P m M); and equation 1 itself serves to calculate how far the base-point must be shifted in order to make the track of the centre of mass assume any given curvature. At the instant that the base-point is placed at a given distance horizontally from the vertical plane containing the centre of mass and its direction of motion, the track of the centre of mass assumes the curvature corresponding to that relative position of the base-point; and in order that the centre of mass may go on moving in a circular track of that curvature, all that remains to be done is to adjust the fore-wheel so that the base-track shall be a circle concentric with the circular track of the centre of mass, as stated in the preceding section, Article 8.

15. *Change of Track from a Straight Line to a Circle.*—In Fig. 5, let the plane of the paper represent a horizontal plane. Let the track of the centre of mass in the first place be a straight line A B, and let it be required to change it to a circle of the radius C e. From what has been stated in the preceding article it is obvious, that the first thing to be done is to incline the fore-wheel in the direction *opposite* to that of the intended curvature, in order that the base-point may be displaced and give rise to centripetal force. The effect of this is to deflect the base-track *away* from the intended centre of curvature C, as shown by the dotted line B D. So soon as the base-track begins to be deflected away from C, the track of the centre of mass begins to become curved towards C, with a curvature which is at first insensible, and increases gradually, as is shown by the plain line B d e. The rider must take care to reverse the direction of the inclination of the fore-wheel plane at the proper moment, in order that the base-track may become curved towards the track of the centre of mass, as shown at D E; and that when the track of the centre of mass has attained its intended steady curvature at the point e, the base-point may be moving in a circle E F, concentric with the circular part $e\,f$ of the track of the centre of mass, and having the deflection e E suited to the curvature. There are thus three positions to be given to the fore-wheel plane: first, a slight inclination opposed to that of the intended curvature, to commence the outward displacement of the base-point, and of the part B D of its track; secondly, an inclination in the same direction with the intended curvature, and somewhat greater than the permanent inclination, to produce the part D E of the base-track, in which it is gradually brought to parallelism with the track of the centre of mass; and, lastly, the permanent inclination suited to produce the part E F of the base-track, which is concentric with the track of the centre of mass. The first two movements occupy a very short time, and are made almost unconsciously by a skilful velocipede rider. The part B d e of the track of the centre of mass, in which it gradually changes from a straight line to a circle, nearly resembles an elastic curve, or the "curve of adjustment" sometimes used in setting out railways, according to a method introduced by Mr. Froude.

16. *The change of Track from a Circle to a Straight Line* is precisely the reverse of that which has been described in the preceding article. It may be represented by supposing the direction of motion in Fig. 5 to be reversed. Then $f\,e$ is the circular track of the centre of mass; C, its centre; F E, the concentric circular base-track, with its proper deflection e E, suited to the curvature and speed. At E, the fore-wheel plane, already inclined inwards, is inclined further inwards, so as to make the base-point describe a sharper curve, E D, gradually approaching the track of the centre of mass. This causes the curvature of the track of the centre of mass to become gradually flattened, as shown at $e\,d$ B. At D the rider inclines the fore-wheel slightly the reverse way, so as to bring the base-track gradually into parallelism with that of the centre of mass, as shown by the dotted curve B D. At B the two tracks are in the same vertical plane; and then the curvature of both ceases, and they become parallel straight lines in one vertical plane.

17. *The change of direction from one Straight Line to another* is made by combining the two operations described in Articles 15 and 16: that is, changing the track first from a straight line to a circle, and then from a circle to a straight line in a different direction from the first. The two straight lines are not exactly tangents to the circle, but pass outside of it: being connected with it by curves of gradually changing curvature, like B d e in Fig. 5.

FIG. 5



18. *Concluding Remarks on Steering.*—The steering, like the balancing, of a velocipede, is analogous to that of a skater; but with this difference, that the skater, when about to describe a circle, at once plants the skate on which he is going to support himself to one side or to the other of the track of his centre of mass, at the proper distance for giving the centripetal force required in order to produce the curvature; and, consequently, his centre of mass begins at that instant to move in a circle of a certain radius; whereas the velocipede-rider has to produce the deflection of the base-track required for the same purpose by degrees, as indicated by the curve B D E in Fig. 5; and, consequently, the curvature of the track of the centre of mass is produced or changed by degrees, as already described.

SECTION III.—PROPULSION.

19. *Resistance on a Level.*—The mathematical principles of the resistance of velocipedes and of the power required for their propulsion are much more simple than those of the balancing and steering; but the experimental data are much less definite and complete.

Let W denote the weight of the rider, and let b be the proportion which the weight of the velocipede bears to that of the rider: then the gross load is $(1 + b)$ W.

The resistance to a velocipede when travelling on a level roadway is caused by the friction of axles and crank-pins, by the roughness of the roadway, and by the impeding action of the air. The resistance caused by friction is independent of the speed. It is directly proportional to the gross load, and inversely proportional to the ratio in which the diameter of a wheel is greater than that of the axles and pins that it carries. The resistance caused by the roughness of the roadway, commonly called *rolling resistance*, is proportional directly to the load, and inversely to the diameter of the wheel. Part of it is independent of the speed, and another part increases with the speed, and, according to Morin, is approximately proportional to the excess of the speed above a certain limit. The resistance of the air at ordinary speeds may be taken as proportional to the square of the speed nearly. In velocipedes the axles and crank-pins are very small compared with their wheels, and are, when kept in proper order, very smooth and well lubricated; the consequence of which is, that the resistance due to their friction must be very small—probably from 0·001 to 0·002 of the load. The principal part of the resistance is the rolling resistance. To represent symbolically the resistance of a velocipede, let f be the *coefficient of resistance:* in other words, the proportion which the resistance bears to the load; let R be the amount of the resistance; then

$$\mathrm{R} = f\,(1 + b)\,\mathrm{W} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The coefficient f is a complex quantity, containing terms independent of the speed, and terms increasing with the speed, terms inversely proportional to the diameter of the wheels, &c. In the absence of precise knowledge of the values of those several terms, various approximate constant values may be taken for the coefficient f, according to the circumstances of the case. No experiments on the resistance of velocipedes have yet come to my knowledge; and in the absence of such experiments we may assume, for purposes of illustration, that the coefficient of resistance of a velocipede is the same with that of a well-made carriage; that is to say (according to Sir John Macneill's experiments), from ·015 to ·03, according to the state of the roadway. In the calculated examples which follow, I will assume $f = 0{\cdot}02$, or $\frac{1}{50}$. Then, if we suppose that the weight of the velocipede is one-fourth of that of the rider (that is $b = \frac{1}{4}$), we shall have

$$\mathrm{R} = \frac{1}{50} \cdot \frac{5}{4} \cdot \mathrm{W} = \frac{\mathrm{W}}{40} \quad . \quad . \quad . \quad . \quad (13\mathrm{A})$$

20. *Resistance on a Slope.*—Let i denote the rate of inclination; in other words, the fraction which expresses when positive the ascent, and when negative, the descent, in the distance unity; then the resistance is expressed as follows:

$$\mathrm{R} = (f \pm i)\,(1 + b)\,\mathrm{W}; \quad . \quad . \quad . \quad . \quad (14)$$

the expression + being applicable to ascents, and — to descents. When there is a descent at a rate expressed by a fraction exceeding the coefficient of resistance f, the excess gives an accelerating force instead of a resistance; and that force is to be counteracted by means of the brake when it tends to make the velocity become too great for safety.

21. *Driving Power and Work on a Level.*—The power required to drive the velocipede is found by calculating the number of units of work done in a given time, such as a second, in overcoming the resistance. Let v be the velocity: then the driving power on a level has the following value:—

$$\mathrm{R}\,v = f\,(1 + b)\,\mathrm{W}; \quad . \quad . \quad . \quad . \quad (15)$$

in other words, the power required in order to drive the velocipede on a level, is equal to that which would *lift the weight of the rider to the following height in each second:*

$$\frac{\mathrm{R}\,v}{\mathrm{W}} = f\,(1 + b)\,v \quad . \quad . \quad (16)$$

The work done in driving the velocipede through *a given distance* x on a level is expressed as follows:

$$\mathrm{R}\,x = f\,(1 + b)\,\mathrm{W}\,x; \quad (17)$$

which is equivalent to the work done in lifting the weight of the rider to the following vertical height:

$$\frac{\mathrm{R}\,x}{\mathrm{W}} = f\,(1 + b)\,x \quad . \quad . \quad (18).$$

Assuming, as before, $f = 0{\cdot}02$, $b = \frac{1}{4}$; we find, for the value of the above expression, $\frac{x}{40}$; that is to say, *the work of driving a velocipede a given distance on a level may be estimated as nearly equal to that of lifting the weight of the rider vertically upwards to a height equal to one-fortieth part of that distance;* in other words, 132ft. of height per mile of distance, or twenty-five metres of height per kilometre of distance. For example, a journey of sixty miles, or 96·56 kilometres, riding a velocipede on a level road, is equivalent to an ascent up a vertical ladder of 7920ft., or 2414 metres high.

22. *Experiments Required.*—In the preceding calculations an assumed value of the coefficient of resistance f is taken, founded upon previous experiments on ordinary wheel carriages. It is desirable that special experiments should be made in order to ascertain directly the coefficients of resistance of velocipedes. The simplest method of making such experiments is the following:—Let the velocipede, mounted by a skilful rider, be started at as high a speed as practicable, on a straight and level roadway. When the full speed has been attained, let the rider remove his feet from the cranks, and allow the velocipede to be gradually retarded by the resistance. Let a series of marks be made in any convenient way (for example, by the rider dropping small pieces of wood) at the points which the velocipede passes, at a series of equal intervals of time (such, for example, as ten seconds), and let the distances between the marks be measured. Let x and x' be two of these consecutive distances, and let t seconds be the length of each of the equal intervals of time; then $\frac{x}{t}$ and $\frac{x'}{t}$ are the mean velocities with which the two distances are described, and $\frac{x - x'}{t}$ is the loss of velocity, or retardation, during the time which elapses between the *middle instants* of the two intervals; that is, during a time equal or nearly equal to t seconds; so that the *rate of retardation* is $\frac{x - x'}{t^2}$, or very nearly so. The rate of retardation produced by a resistance equal to the weight of the moving mass is g = 32·2ft. per second = 9·81 metres per second, nearly; therefore the ratio borne by the actual resistance to the weight of the whole moving mass is given, exactly or approximately, by the following formula:

$$f = \frac{x - x'}{g\,t^2} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The result of Equation 19 is exact if the resistance is constant at all speeds: approximate if the resistance varies with the speed. Similar calculations should be made for different pairs of intervals during the experiment, in order to ascertain whether and how the coefficient of resistance varies with the speed; and similar experiments should be made on roadways in different conditions. Experiments might also be made on the same principle with the velocipede running ronnd circles of different radii, in order to ascertain whether the curvature of the track affects the resistance.

23. *Driving Power and Work upon Inclined Planes.*—In calculating the power expended by the rider in ascending an inclined plane, it is only necessary to put $f + i$ in the place of f in the formulæ; i being the rate of ascent. In calculating the power expended by the rider in *descending* an inclined plane whose rate of inclination is not steeper than that expressed by the coefficient of resistance f (that is to say, according to the preceding estimate, not steeper than 1 in 50), $f - i$ is to be put in place of f. When the rate of descent is equal to or greater than the coefficient of resistance the power expended by the rider is *nothing.*

(*To be continued*).

THE Paterson locomotive works are unusually busy just now, keeping nearly 2000 workmen constantly employed in the three different establishments, severally known as the Rogers, the Danforth, and the Grant Works. During the first six months of this year the Grant Works turned out fifty-two engines, and the other two establishments built about 110, so that Paterson is now turning out engines at the rate of 320 a year, or more than one a day for every working day in the year.