

correct for the rear part of his simplified model, but not for the front unless trail vanishes. Also, (b) he has left out the lateral offset of the front and rear mass center from the track line due to steer angle; this too is correct for the rear part of his simplified bicycle but not for the front unless trail vanishes. (It also appears that he should have included a vertical reaction force at the steering bearing, though this would cancel when (15) and (16) are added.) Finally (c) his centrifugal forces (such a  $f_1$  are in error because he assumes a steady curve due to steer angle divided by a finite wheelbase, whereas in fact even with an infinite wheelbase the *rate* of steer can produce path curvature of the front wheel and with nonzero trail the rate of steer also affects the yaw rate of the rear wheel. Based on these observations, it seems likely that his lean equation could apply correctly to his simplified model only when the trail is zero.

We believe the steer equation could be is formed by adding  $(1 + \frac{e}{l})(eq. 17) + (\frac{e}{l})(eq. 18)$  to eliminate  $P$  (the term multiplied by his  $\ddot{\theta}$ ), but we have not checked this in detail.

*Pearsall, 1922*

In 1922 Pearsall, with the stated intention of extending Bower's [1915] ideas and discovering the **cause** of "speedmans wobble," derived a set of equations for a bicycle model somewhat similar to the Basic bicycle model presented in Chapter 111. He never states precisely whether his model is restricted in any way, but for example, his equations don't include any product of inertia terms, so they are probably not

general.

His technique for deriving the equations of motion was to first linearize the equations of motion of a rolling hoop and then “add on” the trailer effects due to the remaining parts of the bicycle using fairly casual arguments. While his brief verbal justifications sound valid, in fact almost no terms in the equations are exactly correct. We did not make the effort to trace his errors, but note that there may have been a major mistake in the kinematical treatment (which is not spelled out very explicitly): the headings  $\gamma$  and  $\theta$  of the rear and front assemblies are defined relative to the track line, but then they appear to be treated as coordinates relative to inertial space in the equations.

We compared his equation (4) to our steer equation and his equation (5) to our lean equation, and found that his equations differ significantly in almost every term when compared to those presented in Chapter III. Kis equations would also disagree with Bower’s if simplified for Bower’s model.

Pearsall does not say if he compared his equations to Bower’s, and he does not refer to any other works.

### *Timoshenko and Young, 1948*

In this textbook on advanced dynamics, Timoshenko and Young derived a nonlinear (large-angle) lean equation for a simplified Basic bicycle model having only a point mass in the rear part of the bicycle, and a steer angle controlled by the rider. Their model neglects wheel inertias, steering axis tilt, trail and front-mass