ON THE STEERING OF THE BICYCLE.

By G. T. McCAW, B.E.

THE complexity of the question of the steering of the cycle has, no doubt, revealed itself to the rider who has given the matter any attention. He finds at once that the speed has a preponderating influence on the steering. The intelligent rider soon learns also that the weight and size of the steering wheel have a large effect. Indeed, one frequently is told that the larger the front wheel of the bicycle the better will the machine steer—a statement which contains only a partial truth. The steering, however, is affected by other considerations not so obvious, as the wheel base and the proportion of total weight on the front wheel. The question then arises, Can a general formula be obtained which will include all the foregoing, and by which the proper design of the steering head may be determined?

It is hardly necessary to point out the necessity for correct steering: every experienced rider knows how much less is the fatigue to hands and arms of riding an easily-steered machine. It is therefore desirable that the causes which affect the question should be investigated, and, if possible, a theoretical analysis worked out, from the results of which the best design can be determined beforehand, thus avoiding the waste of time and money entailed by the alternative necessity for a series of experimental tests on the various models.

Since the straight front fork was abandoned in favour of the curved, opinions have varied as to the proper design of the latter. Each maker has his own ideas on the subject, and only in a few cases does success seem to be attained. Thus the variation of the rake—the angle which the steering head makes with the vertical—is considerable. The perpendicular distance from the centre of the front wheel on the steering axis—*i.e.*, the axis of the steering socket—is also very variable. Again, the amount of curvature of the front fork frequently exhibits a difference of opinion. Some consideration should reveal the fact that provided the rake and the perpendicular above mentioned are the same, the particular curvature has no effect on the steering, though for purposes of strength the curvature should be as continuous as possible, and ought not to change abruptly, as in some models of to-day. It is obvious that if the rake and perpendicular distance above are determined, the position of the steering axis is determined; or the axis may be fixed by finding the rake and the point where the axis cuts the line of ground contact.

The following investigation necessarily involves a rather complex analysis; the results are, however, summed up at the end for those who do not care to wade through the mathematics of the subject.

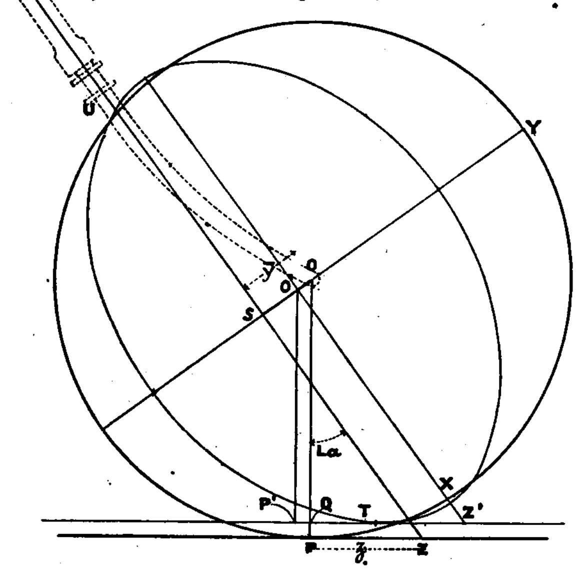
We shall assume, at the beginning of the argument, that the bicycle is moving at a uniform rate on a circular track, and that the banking is such that the surface is perpendicular to the plane of the machine. The resultant, therefore, of the weight and centrifugal force is perpendicular to the surface of the track. In what follows we shall speak of this as the "resultant," and when we speak of a rise or fall of the centre of gravity it is to be understood that the distance is measured on this resultant and

From this equation it follows that $h = o$ when (1) $\alpha = o$, and (2) when

$$\sin \alpha = \frac{a\,y}{a^2 - (a^2 - r^2)\sin^2\frac{\theta}{2}},$$

or $\sin \alpha = \frac{y}{a}$ approximately, when θ is small. Since under these circumstances $z = o$, we have proved the

Fig I.



following, which is, of course, otherwise evident. When the steering axis passes through the point of ground contact, h is approximately zero for small values of θ. If there were no gyrostatic action, therefore, the steering axis should intersect the lowest point of the wheel, and this is nearly the correct position when the speed of the cycle is very low.

From equation (B) the following is derived:—

$$d\,h = \frac{\sin\alpha\sin\theta\,d\,\theta}{\sqrt{1-\sin^2\alpha\sin^2\theta}}\left[a\sin\alpha\cos\theta - y\sqrt{1-\sin^2\alpha\sin^2\theta}\right] \quad . \quad . \quad . \quad . \quad (C).$$

In the second place, it is necessary to find a value for K, the gyrostatic moment. If ω be the angular velocity of rotation of the fore wheel, ϕ its rate of turning round the track, or, as it is called, the angular velocity of precession, then the above quantities are connected by the following equation:—

$$K = \frac{w}{g} \;.\; k^2\,\omega\,\phi \quad . \quad . \quad . \quad . \quad (D)$$

—Perry: "Spinning Tops," p. 90.

Now $\phi = \frac{v}{r}$, and r may be found as follows. Referring to Fig. 2, L Z is the trace of the front wheel as the

friction of the steering head, (2) the ground resistance to rolling, (3) the ground resistance to turning the wheel. Of these the first two need not be considered. The third, however, may not be negligible if the speed is low and the tire soft; however, as the speed increases the friction will diminish, and in the absence of experiments it is doubtful whether this resistance would appreciably affect the result at ordinary speeds with tires properly inflated. It has also been assumed that the speed is constant. Especially in the case of high gears this is not quite true; obviously, however, the machine should be designed for the average speed on the level.

We shall now discuss the practical results which this formula (G) affords. In the first place, it allows a choice of rake, but the angle of rake having been chosen, the position of the steering axis is absolutely fixed, since the formula gives a definite value for z—the distance from point of front wheel contact with base line to the point where the steering axis cuts the ground—corresponding to any chosen value of α, the angle of rake. The choice of this angle, within certain limits, depends on other considerations than the steering, such as strength, stiffness, and general convenience of design. We say within certain limits, because, suppose the angle were excessively increased so that the spring of the front forks became considerable, it is obvious that this in itself would adversely affect the steering; indeed, it may be stated generally that any diminution of strength and stiffness of the fore-carriage will militate against ease of steering, provided, of course, that the position of the axis is correct in the first instance. As an example of the influence of the question of strength in deciding the proper angle, it may be noticed that this angle should be as far as possible chosen that y—the perpendicular distance from the centre of front wheel on the steering axis—may be as small as possible, for it is evident that the greater the distance through which the centre of the front wheel is moved from the plane of the machine, the greater will be the cross strains introduced, and consequently the more will the strength be impaired. With the ordinary safety the limits above mentioned may be 18 deg. and 25 deg.

In the second place, the formula shows the great influence of speed on the steering, for it proves that z increases as the square of the velocity. Again, it demonstrates that the distance z should increase directly as the weight of the front wheel, and approximately as the radius of the same. These three results are conformable to experience. The formula, however, also proves that z should decrease as the proportion of total weight on front wheel, the wheel base, and the rake increase. These latter results, though they follow from the argument, are not so obvious to the expert rider. We shall apply this formula to a few examples.

Example 1.—Taking the case of a safety having the dimensions already given, and supposing the rake to be about 24½ deg., so that $\sin \alpha = \frac{5}{12}$, it follows that z should be 3·82in.

Example 2.—If in the same safety the speed were only eight miles an hour = 12ft. per second, the distance z would be only 2·12in.

Example 3.—If the machine were a tandem—weight of front wheel, 4½ lb.; radius of gyration, 12½in.; wheel base, 66in.; and speed, 18ft. per second (12½ miles per hour);

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W weight of machine and rider, 190 lb.
w weight of front wheel, 4 lb.
R resultant of total weight and centrifugal force.
n fraction of resultant weight on front wheel, $\frac{4}{15}$.
a radius of front wheel, 14in.
k radius of gyration of front wheel, $12\frac{5}{16}$in.
b wheel base, 44in.
v velocity on track, $10\frac{1}{4}$ miles per hour = 15ft. per second.
r radius of track.
α rake of steering head.
angle through which the handle-bar is turned.
z the distance from point of ground contact of front wheel to point where steering axis cuts the base.
y length of perpendicular on steering axis from centre of front wheel.

The steering axis is now usually designed to cut the ground in front of the point of wheel contact. In this case a rotation of the handles will have the effect of lowering the centre of the front wheel by an amount we shall call h. Hence, when the cycle is moving round the track, the plane of the front wheel making an angle θ with the plane of the machine, the centre of gravity of machine and rider—considered as one system—will have fallen by an amount $n\,h$. The resultant tends to move the steering wheel further from the plane of the machine; its effect, however, is neutralised by the gyrostatic couple, and correct steering will be assured by equilibration of these opposing forces. We have, therefore, by the principle of work,

$$R\,.\,\delta\,(n\,h) = K\,.\,\delta\,\theta \quad . \quad . \quad . \quad (A),$$

where K is the gyrostatic couple. It will first be necessary to find h.

In Fig. 1 the steering socket is supposed held rigidly in position while the wheel is turned through an angle θ. The wheel is now raised off the ground by the amount h. If the wheel be projected back on the normal plane its figure will be an ellipse, centre O′, and the height of the horizontal tangent to this ellipse will give the required distance.

The equation of the ellipse to the axes is

$$x^2 \cos^2\theta + y^2 = a^2 \cos^2\theta.$$

The equation of the horizontal tangent P′ Y is

$$x \cos\alpha - y \sin\alpha = a\sqrt{1 - \sin^2\alpha \sin^2\theta}.$$

The height of O′ above the base line P Z is

$$a - y\,(1 - \cos\theta)\sin\alpha$$

Hence

$$h = a - y \sin\alpha\,(1 - \cos\theta) - a\sqrt{1 - \sin^2\alpha\sin^2\theta} \quad (B).$$

of rotation of the fore wheel, ϕ its rate of turning round the track, or, as it is called, the angular velocity of precession, then the above quantities are connected by the following equation:—

$$K = \frac{w}{g}\,.\,k^2\,\omega\,\phi \quad . \quad . \quad . \quad . \quad (D)$$

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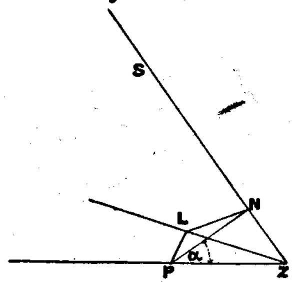
Now $\phi = \frac{v}{r}$, and r may be found as follows. Referring to Fig. 2, L Z is the trace of the front wheel as the

Fig 2.



machine travels round the track. If, then, a plane L N P be drawn perpendicular to the steering axes S Z, the angle L N P = θ. Hence when θ is small

$$\theta = \frac{P\,L}{P\,Z\cos\alpha} = \frac{\angle\,P\,Z\,L}{\cos\alpha}.$$

Also, it is evident that $\angle\,P\,Z\,L = \frac{b}{r}$.

Hence $\phi = \frac{v\;\theta\;\cos\alpha}{b}$. Also $\omega = \frac{v}{a}$. Substituting these values of ϕ and ω in equation (D),

$$K = \frac{k^2}{a\,b}\,.\,\frac{w\,v^2}{g}\,.\,\theta\cos\alpha \quad . \quad . \quad . \quad (E).$$

Having thus found expressions for K and $\delta\,h$, we apply these to equation (A), which then becomes, when the increment of θ is taken infinitely small—

$$n\,.\,R\,.\,\frac{\sin\alpha\sin\theta}{\sqrt{1 - \sin^2\alpha\sin^2\theta}}\,\Big[a\sin\alpha\cos\theta -$$

$$y\sqrt{1 - \sin^2\alpha\sin^2\theta}\Big] = \frac{k^2}{a\,b}\,.\,\frac{w\,v^2}{g}\cos\alpha\,.\,\theta \ (F).$$

From this equation y can be found for all values of α.

Suppose, however, that θ is very small, so that the cycle is travelling practically in a straight line, then neglecting $\sin^2\theta$ and putting $\sin\theta = \theta$ and $\cos\theta = 1$, the equation (F) becomes

$$n\,W\sin\alpha\,(a\sin\alpha - y) = \frac{k^2}{a\,b}\,.\,\frac{w\,v^2}{g}\,.\cos\alpha.$$

But $a\sin\alpha - y = z\cos\alpha$, hence

$$z = \frac{w\,.\,v^2\,.\,k^2}{n\,W\,.\,g\,.\,a\,b\sin\alpha} \quad . \quad . \quad . \quad (G).$$

This neat result gives the value of z corresponding to any chosen rake of head, and hence the position of the steering axis is exactly determined.

In the above investigation we have neglected (1) the

These latter results, though they follow from the argument, are not so obvious to the expert rider. We shall apply this formula to a few examples.

Example 1.—Taking the case of a safety having the dimensions already given, and supposing the rake to be about $24\frac{1}{2}$ deg., so that $\sin\alpha = \frac{5}{12}$, it follows that z should be 3·32in.

Example 2.—If in the same safety the speed were only eight miles an hour = 12ft. per second, the distance z would be only 2·12in.

Example 3.—If the machine were a tandem—weight of front wheel, $4\frac{1}{2}$ lb.; radius of gyration, $12\frac{1}{2}$in.; wheel base, 66in.; and speed, 18ft. per second ($12\frac{1}{4}$ miles per hour); weight on front wheel, 130 lb.—then, other things being the same, the value of z would be 1·84in.

The following table gives the values of y and z for a single safety of the dimensions stated at beginning of article, the steering angles varying from 15 deg. to 35 deg.:—

α	15°	18°	20°	22°	24°	25°	26°	28°	30°	35°
	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
y	−1·53	+0·08	1·00	2·20	2·60	2·96	3·31	3·98	4·61	6·06
z	+5·33	4·46	4·08	3·68	3·39	3·26	3·14	2·93	2·76	2·40

In the same way tables may be formed for any speed, size or weight of wheels, weight of rider and machine, &c., and for any type of rear-driven safety or tandem bicycle. The results are also approximately true for tricycles of the Cripper type.

AMERICAN PADDLE-WHEEL STEAMERS WITH BEAM ENGINES.

No. VI.

The frequency with which the Hudson River steamers have been lengthened has already been commented upon, and the lengthening of the New York by 30ft. in 1897, by cutting it apart amidships, and pulling the bow section ahead 30ft., to make room for a new section, was an operation of exceptional interest. The following particulars are therefore given somewhat in full, from information published in the *Scientific American*, and from drawings furnished the writer by Mr. W. D. Dickie, Superintendent of the Erie Basin Dry Docks, owned by the John N. Robins Company. The New York was taken into the dry dock at Brooklyn, and was placed upon the ordinary arrangement of keel blocks and bilge blocks. The vessel was then cut between the engine and the boilers, the hull being cut at the riveted joints, and the superstructure being cut straight across.

As the forward part was to be pulled ahead, launching ways and cradles were built under this part of the boat. The ways were 102ft. long, placed 10ft. 9in. apart in the clear, and outside of these main ways—3ft. 9in. from them—were additional ways to take the weight of the boilers. The fixed ways were of timbers 10in. deep and 14in. wide, laid upon blocking upon the floor of the dock. Guide pieces 12in. deep were spiked against the outside of these, to keep the sliding ways in place. These sliding ways were timbers, 6in. by 14in., laid flat, and connected at regular intervals by transverse timbers, 12in. by 12in., resting upon them. The contact surfaces of the fixed and sliding ways were planed and smoothed, and were