equations of motion. Dohring refers to S & K, but never states explicitly how his equations compare.

Collins, 1963

In his 1963 University of Wisconsin Ph.D. dissertation R. N. Collins, working on a project supported by Harley Davidson Motor Company, studied a Basic bicycle model with the addition of a driving force on the rear tire and an explicit force for aerodynamic drag applied to the front fork/handlebar assembly. He derived the equations of motion using Euler's equations (Newton's Laws) for the 4 rigid bodies of the Basic bicycle model.

Collins derives nonlinear velocity and acceleration expressions for the rear and front center of mass first (see pages 19 and 20 of his dissertation), and then linearizes about the vertical equilibrium position, before deriving the linearized equations of motion. By writing the drive force and aerodynamic drag force as a function of the square of the forward velocity of the motorcycle (see p. 12 in his dissertation), he alters the vertical contact forces on the front and rear wheels. By making the assumptions of no slip angle and constant velocity he has only two degrees of freedom for his model and he is therefore able to write the linearized governing equations as two coupled second order ordinary differential equations in the lean and steer angles (see p. 76 eq. (5.1) and eq. (5.2) in his dissertation). The final equations are complicated in appearance and include over 30 quantities defined in terms of motorcycle parameters. (These quantities often include previously defined quantities, which further complicates understanding of the equations.)

His equation (5.1) is not exactly the steer equation, and his equation (5.2) is not exactly the lean equation. However, if we transfer all the terms to the left hand side, and form the combination,

$$\sin\alpha[eq.(5.1)] + h_2[eq.(5.2)] = [equation\ with\ no\ \dot{\phi}\ \textbf{and}\ no\ M_3] = 0,$$

the result appears to be the lean equation. That is, in our notation the coefficients $\mathcal{M}_{\chi\chi}$, $\mathcal{C}_{\chi\chi}$ (which is zero), $\mathcal{K}_{\chi\chi}$ are all in agreement with those presented in Chapter 111. The steering moment M_3, **our** equivalent M_ψ, also drops out of the equation as it should. So while the task of multiple substitution **was** tedious and prevented us from completely comparison of the lean equation, or even from determining what cornbination of his equations ought to give **our** steer equation, it may be that Collins' resulting equations are correct.

The only potential flaw to come to light is that Collins' equivalent to our $\mathcal{C}_{\chi\psi}$ term, namely

$$-(\sin\alpha K_{21} + h_2 K_{31}),$$

should probably include the angular momentum of both wheels. However, this expression appears to contain only the front moment of inertia I_1', not I_2'.

Collins refers to the **works** of Sommerfeld and Klein [1903], Bower [1915], Pearsall [1922], and Döhring [1955], but never compares his equations to theirs (nor to those of Whipple [1899] or Carvallo [1901], who were cited by S & K).