

Overall, the definitions of Whipple's variables are difficult to decipher and make his paper difficult to read, but his equations appear to be rigorously derived and are fully general when compared to those given in Chapter 111. Whipple is critical of McGaw's [1898] study of tricycles, and Bourlet's [pre-1896] study of bicycles, neither of which have we read.

Carvallo, 1901

Carvallo [1901] wrote 300 generally lucid pages on the stability of monocycles (rider inside a single wheel) and bicycles. Only the second part of the monograph, which won a prestigious prize, concerns us. In it he modifies Lagrange dynamics to deal with rolling hoops and bicycles (we were not able to tell if his method is a different way of dealing with nonholonomic constraints). We are concerned primarily with section V on no-hands stability. The equations where each term was derived are laid out on pp. 100-101, and restated in condensed form on p. 103. The equations are exactly analogous to ours, one for lean and one for steer.

Although we could not find where Carvallo said this, it appears that his bicycle has two identical heavy wheels, the rider and frame are considered a single unit, and the mass of the front assembly is at the center of the front wheel and its inertia properties are those of the wheel. (This is not an unreasonable idealization if the handlebars are not heavy and are positioned on the steering axis as was common in designs of that day. Technically, for such a design the mass of the handlebars and straight part of fork can then be considered part of the rear frame.)

We find that Carvallo's equations (for a bicycle with massless forks and handlebars) agree exactly with ours. Most quantities are defined in the text, but the reader should note that the wheel inertias are defined relative to their ground contact, i.e. C_1 is for spin about a diameter, A_1 is for lean (i.e., $A_1 = C_1 + \mu_1 R^2$), B_1 is for rolling about the contact point (i.e., $B_1 = I_p + \mu_1 R^2$). $S = \frac{V}{R}$ is the wheel rotation rate. Carvallo makes no reference to other works, which is not surprising as his research was evidently performed in 1898.

Sommerfeld and Klein, 1903

Sommerfeld and Klein (S & K) in 1903 derived the linearized equations of motions for the Basic bicycle model having all the mass and inertia of the front assembly in the front wheel (similar to Carvallo). Somewhat similar to Whipple [1899], they used a Newtonian analysis of the front and rear assembly, and treated the two parts as two trailers attached to the steering axis, deriving the linearized equations of motions using axes parallel to the steering axis. S & K refer to Whipple [1899] and Carvallo [1901] but do not say whether their equations agree.

Their equations are most easily compared to Döhning's [1955], and are found to be a correct subset of his. It is possible that S & K's slight simplification(s) to the model were due to their main interest in determining what effect the wheels as gyroscopes had on the stability (since the article is a chapter in their massive work on gyroscopes). They are critical of Bourlet [1898] (whose book we have not read).