Bower, 1915

Bower in 1915, without reference to any previous bicycle work, derived the linearized equations of motion for a simplified Basic bicycle model at the end of an article mainly concerning the gyroscopic effects of the engine and wheels on steady turns. His model consists of a rear frame with its center of mass above the rear contact point, having polar inertia $R_{\overline{zz}}$ provided by two point masses, one ahead and one behind the center of mass. Two smaller masses at the same height are attached to the front assembly. Wheel inertia and caster trail are also included, but the steering axis is restricted to be vertical.

Instead of providing two second order equations for his model, he presents the governing 4th order linear differential equation (eq. (19) in his analysis), which is not convenient for comparison. The e coefficient, given as equation (24) in his paper, is comparable to the determinant of the $\mathcal{K}$ matrix presented in our Chapter III. Comparison indicates that Bower's equations must be missing the $g\nu$ term in the $\mathcal{K}_{\chi\psi}$ coefficient of the lean equation for his simple model, which confirms that his equations lack some of the effects of trail has on the bicycle. No comparison was made to Bower's coefficients A-D for his simplified bicycle model, but casual observation indicates they also lack terms.

Looking back at his derivation it appears that his ϕ is our $-\psi$, and his θ is our χ_r. His eqs. (15) and (16) may be added to eliminate the internal reaction P, thus leading to a lean equation. However, (a) he has ignored product of inertia terms (relative to the wheel contacts) which should appear multiplying his $\ddot{\psi}_1$, $\ddot{\psi}_2$; this is correct for the rear part of his simplified model, but not for the front unless trail vanishes. Also, (b) he has left out the lateral offset of the front and rear mass center from the track line due to steer angle; this too is correct for the rear part of his simplified bicycle but not for the front unless trail vanishes. (It also appears that he should have included a vertical reaction force at the steering bearing, though this would cancel when (15) and (16) are added.) Finally (c) his centrifugal forces (such a f_1 are in error because he assumes **a** steady curve due to steer angle divided by a finite wheelbase, whereas in fact even with an infinite wheelbase the *rate* of steer can produce path curvature of the front wheel and with nonzero trail the rate of steer also affects the yaw rate of the rear wheel. Based on these observations, it seems likely that his lean equation could apply correctly to his simplified model only when the trail is zero.

We believe the steer equation could be is formed by adding $(1 + \frac{c}{l})(eq.\ 17) + (\frac{c}{l})(eq.\ 18)$ to eliminate P (the term multiplied by his $\ddot{\theta}$), but we have not checked this in detail.

Pearsall, 1922

In 1922 Pearsall, with the stated intention of extending Bower's [1915] ideas and discovering the **cause** of "speedmans wobble," derived a set of equations for a bicycle model somewhat similar to the Basic bicycle model presented in Chapter 111. He never states precisely whether his model is restricted in any way, but for example, his equations don't include any product of inertia terms, so they are probably not