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Motion Analysis of A Motorcycle Taking Account of Rider's Effects

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ABSTRACT

In this paper, to analyze rider's effects on the motion of a motorcycle, we model a rider-motorcycle system taking account of the leaning motion of the rider's upper torso and the rider's arm. In addition, the nonlinearity of the tire force is introduced to the tire model taking account of the cross-sectional shape, the elastic deformation and the tire-ground contact area. On the basis of the derived nonlinear state-space model, we analyze the effects of not only the rider's arm but also his/her postures in steady-state turning by simulations. The rider's postures of lean-with, lean-in, and lean-out are realized by adding the lean torque to the rider's upper torso. The motorcycle motion and the rider's effects are analyzed in the case where the friction coefficient of the road surface changes severely in the steady-state turning. Also, the linearized steady-state turning model is derived, and the stability analysis of the motorcycle in the steady-state turning is performed.

Keywords: modeling, rider-motorcycle system, lean-in/lean-with/lean-out, motion analysis, stability analysis.

1 INTRODUCTION

A rider can realize stable running of a motorcycle under his/her driving movement including steering operation, braking, torso movement, and others. The rider's effects should be taken into account for investigation of the motorcycle motion analysis. Although it may be very difficult to measure the rider's driving movement by experiments, the use of a dynamical model of the rider-motorcycle system makes it possible to easily analyze the rider's driving movement and avoid the rider's risk associated with the experiments. These analyses are also greatly useful for the design of motorcycles, which enhances the maneuverability and stability of the motorcycles.

A number of researchers have presented some dynamical models to simulate and analyze the rider's driving movement and the motorcycle motion. In 1971, Sharp [1] presented a linear model with four degrees of freedom of the yaw angle, the roll angle, the steering angle, and the lateral velocity, and analyzed the motorcycle stability in the straight running. The rider was rigidly attached to the rear frame in this model. After that, various models were presented taking account of more degrees of freedom and more parts of the motorcycle [2], [3]. In 1982, Koenen and Pacejka [4] developed a dynamical model which allows the rider's upper torso to have leaning motion restrained by rotational spring and damper, and analyzed influences of the rider's upper torso leaning on free vibrations of motorcycles in curves. In 1984, Kageyama and Kogo

[5] replaced the rider's arm that gripped the handlebars with spring and damper elements. The differences of the rider's handle grip and press forces were simulated and investigated by changing the spring and the damper. In 1988, Katayama et al. [6] added two degrees of freedom of the rider's upper torso and lower torso to the Sharp's motorcycle model [1], and presented a rider driving model. With advancement of the computer technology, a number of researchers carried out simulations of motorcycles using some commercial dynamics analysis software. It has been shown that the simulation results obtained by using the commercial software are in fair agreement with the responses of a real motorcycle [7]-[9].

The authors presented a nonlinear dynamical model of a motorcycle based on multibody dynamics theory [10], [11], and further developed a model of the rider-motorcycle system by taking account of the leaning motion of the rider's upper torso [12]. In the dynamical model, the rider's upper torso was not connected to the handlebars, and the motorcycle motion differed from that of the actual motorcycle in the effect of the rider's arm.

In this study, we model the rider-motorcycle system taking account of not only the leaning motion of the rider's upper torso but also his/her arm. A nonlinear state-space model and the linearized steady-state turning model are derived. On the basis of the nonlinear state-space model, the effects of not only the leaning motion of the rider's upper torso but also his/her arm in the steady-state turning are analyzed by simulations. Using the linearized steady-state turning model, the stability analysis of the motorcycle is performed.

2 MODELING A RIDER-MOTORCYCLE SYSTEM

2.1 Dynamical model

A dynamical model of a rider-motorcycle system is shown in Figure 1. The model is divided into four rigid bodies: the rear frame (comprising rider's lower torso, main frame, rear fork, tank, engine, etc.), the front frame (comprising handlebars, steering shaft, front fork, etc.), the rear wheel, and the front wheel, and these are connected by three revolute joints [10], [11]. The rider's upper torso is connected to the rear frame by a rotational spring K_{wx} and a rotational damper C_{wx} [12], [13]. This makes the rider's upper torso have one degree of freedom around the roll axis (i.e., lean angle). In addition, we take into account the mass of the upper torso, and the moment of inertia about the roll axis and the yaw axis.

Referring to the proposal about connecting the rider's upper torso and the handlebars [5], the rider's upper torso is connected to the handlebars by a spring K_{wz} and a damper C_{wz}. Namely, the rider's upper torso receives the reaction torque to the steering torque added to the handle axis via the arm consisting of the spring and the damper. The steering torque τ_{fr} from the rider is directly added to the handle axis. The symbols used in Figure 1 are as follows. *W*: center of mass of the rider's upper torso, *A*: center of mass of the rear frame, *U*: center of mass of the front frame, *C*: center of mass of the rear wheel, *D*: center of mass of the front wheel, m_W: mass of the rider's upper torso, m_A: mass of the rear frame, m_U: mass of the front frame, m_C: mass of the rear wheel, m_D: mass of the front wheel, K_{wz}: spring, C_{wz}: damper, K_{wx}: rotational spring, C_{wx}: rotational damper, R_r: rear wheel radius, R_f: front wheel radius, τ_{fr}: steering torque from the rider, τ_{wx}: lean torque controlling the rider's upper torso, τ_r: rear wheel driving torque. Table 1 shows specifications of the rider-motorcycle system, referring to type C specifications of JSAE Technical Report Series 25 [14].

In Figure 1, the O coordinate system is the inertial coordinate system, and the *A*, *C*, *D*, *U*, *W* coordinate systems are the standard coordinate system of the rigid bodies. The generalized coordinate $\boldsymbol{Q}$ of the rider-motorcycle system consists of the position of the center of mass of the rear frame $\boldsymbol{R}_{OA}$, the Euler angle of the rear frame $\boldsymbol{\Theta}_{OA}$, the steering angle δ, the lean angle of the rider's upper torso θ_{wx}, the rotation angle of the rear wheel θ_r, and the rotation angle of the front wheel θ_f.

$$\boldsymbol{Q} = [\boldsymbol{R}_{OA}^T \quad \boldsymbol{\Theta}_{OA}^T \quad \delta \quad \theta_{wx} \quad \theta_r \quad \theta_f]^T \tag{1}$$

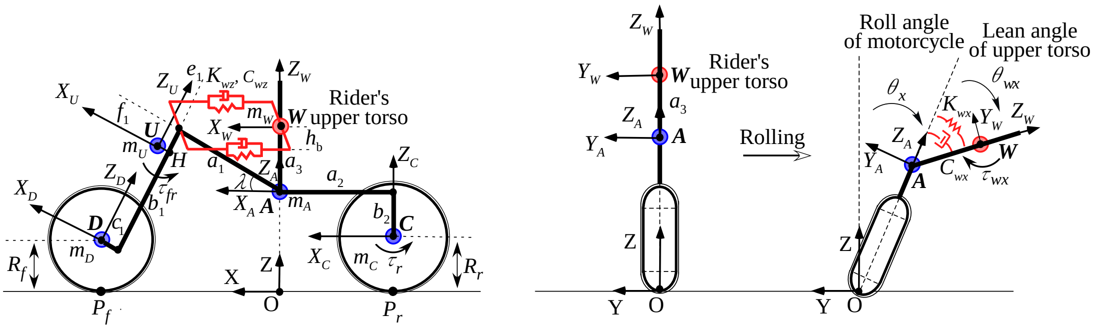
The generalized velocity $\boldsymbol{S}$ is

$$\boldsymbol{S} = [\dot{\boldsymbol{R}}'^{T}_{OA} \quad \dot{\boldsymbol{\Theta}}^{T}_{OA} \quad \dot{\delta} \quad \dot{\theta}_{wx} \quad \dot{\theta}_{r} \quad \dot{\theta}_{f}]^{T} \tag{2}$$

where the superscript T indicates the transpose. A variable without a superscript dash is described in the inertial coordinate system O, and a variable with the dash indicates that it is described in the standard coordinate system for each rigid body.

The relationship between the generalized coordinate $\boldsymbol{Q}$ and the generalized velocity $\boldsymbol{S}$ can be expressed as follows.

$$\dot{\boldsymbol{Q}} = \frac{\partial \dot{\boldsymbol{Q}}}{\partial \boldsymbol{S}} \boldsymbol{S} = \begin{bmatrix} \boldsymbol{C}_{OA} & \boldsymbol{O}_{3\times 7} \\ \boldsymbol{O}_{7\times 3} & \boldsymbol{I}_{7} \end{bmatrix} \boldsymbol{S} \tag{3}$$



(a) Side view (b) Rear view

Figure 1. A dynamical model of the rider-motorcycle system

Table 1. Specifications of the rider-motorcycle system

Mass (kg)	m_A	m_U	m_W	m_C	m_D
	164.43	15.50	50.00	19.20	10.90
Inertia (kgm^2)	I'_{OAxx}	I'_{OUxx}	I'_{OWxx}	I'_{OCxx}	I'_{ODxx}
	26.04	1.74	4.75	0.41	0.26
	I'_{OAyy}	I'_{OUyy}	I'_{OWyy}	I'_{OCyy}	I'_{ODyy}
	24.73	0.30	0.00	1.68	0.47
	I'_{OAzz}	I'_{OUzz}	I'_{OWzz}	I'_{OCzz}	I'_{ODzz}
	26.28	0.40	4.75	0.41	0.26
Length (m)	a_1	a_2	a_3	b_1	b_2
	0.5447	0.5231	0.3586	0.7068	0.3070
	c_1	f_1	e_1	R_r	R_f
	0.0503	0.1298	0.0490	0.3120	0.2990
Length (m)	h_b		Angle (deg)	λ	
	0.3			27	
Spring stiffness of rider's upper torso			K_{wx}	K_{wz}	
			350 (Nm/rad)	172.2 (N/m)	
Damping coefficient of rider's upper torso			C_{wx}	C_{wz}	
			20 (Nms/rad)	26.4 (Ns/m)	

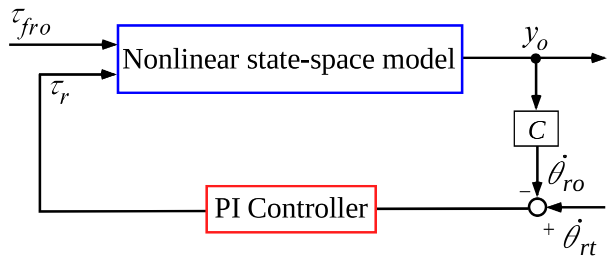


Figure 2. Closed-loop control system for obtaining the equilibrium points of the steady-state turning

where $\boldsymbol{C}_{OA}$ is the rotation matrix from the rear frame coordinate system A to the inertial coordinate system O. In addition, we introduced the nonlinearity of the tire force [15] to the tire model taking account of the cross-sectional shape, the elastic deformation and the tire-ground contact area [11].

2.2 Nonlinear state-space model

Based on motion analysis of each rigid body, adding constraints such as the revolute joints, and using velocity conversion [10], [11], [16], the nonlinear equations of motion of the rider-motorcycle system are obtained.

$$\boldsymbol{m}^S \dot{\boldsymbol{S}} = \boldsymbol{f}^S \tag{4}$$

where $\boldsymbol{m}^S$ and $\boldsymbol{f}^S$ are the mass matrix and the force matrix of the rider-motorcycle system.

Using Equations (3) and (4), the nonlinear state-space model is derived.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{E}(\boldsymbol{x}) \tag{5}$$

where the state variable $\boldsymbol{x}$ consists of the generalized coordinate $\boldsymbol{Q}$ and the generalized velocity $\boldsymbol{S}$, and the input $\boldsymbol{u}$ consists of the steering torque from the rider τ_{fr}, the lean torque τ_{wx} and the rear wheel driving torque τ_r. The rider's upper torso receives the reaction torque to the steering torque added to the handle axis via his/her arm, consisting of the spring K_{wz} and the damper C_{wz}. The lean torque τ_{wx} is the torque that controls the leaning motion of the rider's upper torso. The rear wheel driving torque τ_r is used to control the vehicle speed.

2.3 Linearized steady-state turning model

The steady-state turning is generated using the derived nonlinear state-space model, and the linearization is performed around the equilibrium points. In Figure 2, in order to obtain the equilibrium points of the steady-state turning, the constant steering torque τ_{fro} from the rider and the rear wheel driving torque τ_r are added into the nonlinear steady-state turning model. The rear wheel driving torque τ_r is calculated by the following control to the target rotational velocity of the rear wheel $\dot{\theta}_{rt}$.

The derived nonlinear state-space model Equations (3) and (4) are linearized about the obtained equilibrium points of the steady-state running as follows.

$$\Delta\dot{\boldsymbol{Q}} = \Delta\frac{\partial\dot{\boldsymbol{Q}}}{\partial\boldsymbol{S}}\boldsymbol{S}_o + \left.\frac{\partial\dot{\boldsymbol{Q}}}{\partial\boldsymbol{S}}\right|_{\boldsymbol{o}}\Delta\boldsymbol{S} \tag{6}$$

$$\boldsymbol{m}_o^S \Delta\dot{\boldsymbol{S}} = \Delta\boldsymbol{f}^S \tag{7}$$

The value at the equilibrium point is indicated with the suffix o, and the small variation from the equilibrium point is indicated by Δ.

From Equations (6) and (7), the linearized steady-state turning model is derived.

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}_l\Delta\boldsymbol{x} + \boldsymbol{B}_l\Delta\boldsymbol{u} \tag{8}$$

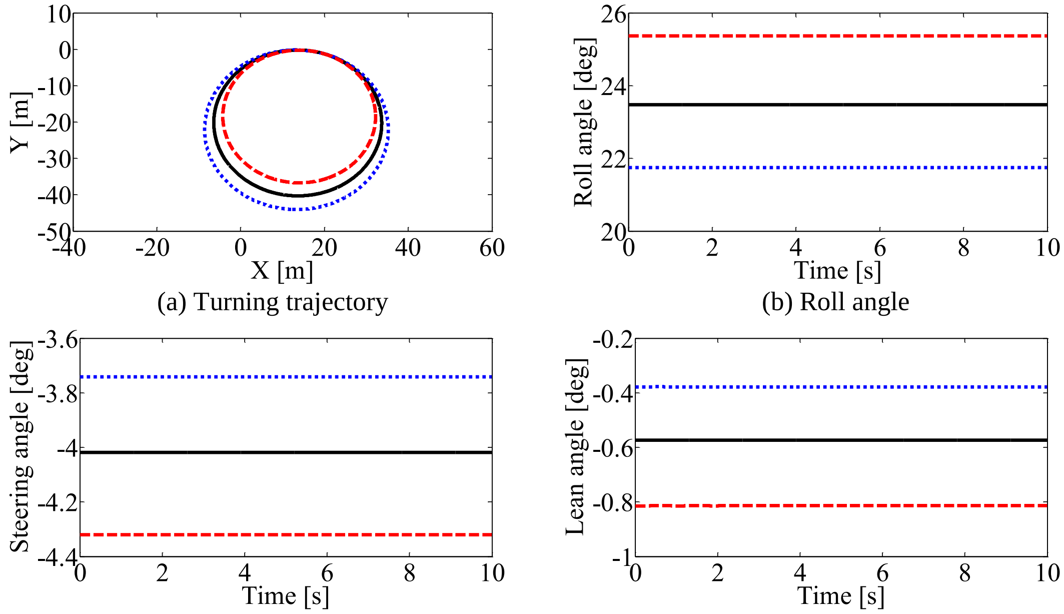
3 MOTION ANALYSIS OF RIDER-MOTORCYCLE SYSTEM IN STEADY-STATE TURNING

On the basis of the closed-loop control system of Figure 2, we perform the steady-state turning simulations using the derived nonlinear state-space model. In this chapter, the vehicle speed is controlled at 35 km/h and the constant steering torque of -9 Nm from the rider is directly added to the handle axis. Also, effects of the rider's arms and postures of lean-with, lean-in, and lean-out are considered in the simulations. In this paper, the posture of lean-with is defined so that the rider's upper torso leans in the same direction of the motorcycle and generates the lean angle within a few degrees to the roll angle of the motorcycle. The posture of lean-in is defined so that the rider's upper torso leans in the same direction of the motorcycle. The posture of lean-out is defined so that the rider's upper torso leans in the opposite direction of the motorcycle. The lean torques of 20 Nm and -20 Nm are added to the rider's upper torso to realize his/her postures of lean-in and lean-out, respectively. Also, stability analysis of the motorcycle is performed for the linearized steady-state turning model.

3.1 Steady-state turning simulations using the nonlinear state-space model

First, we investigate the effect of the rider's arm on the motorcycle motion in the steady-state turning when the rider keeps the posture of lean-with. In Figure 3, the solid lines indicate simulation results in the case of the rider's arm simulated by the spring K_{wz} and the damper C_{wz}, the dotted lines indicate simulation results in the case of the spring stiffness and the damping coefficient of the arm increased to $2K_{wz}$ and $2C_{wz}$, and the broken lines indicate simulation results for the model without the rider's arm. (a), (b), (c) and (d) show the turning trajectory, the roll angle, the steering angle, and the lean angle of the rider's upper torso, respectively.

In the case of the rider's arm simulated by the spring K_{wz} and the damper C_{wz}, the steering torque of -9 Nm from the rider holds the steering angle at about -4 deg, and in the sequel the steady-state turning with the radius of 20 m and the roll angle of about 23.5 deg are realized. In the case of the model without the rider's arm, the steering angle and the roll angle increase by about -4.3 deg and 25.4 deg respectively, and the turning radius becomes about 18 m. In the case of the rider's arm with $2K_{wz}$ and $2C_{wz}$, the steering angle and the roll angle decrease by about -3.7deg and 21.8 deg respectively, and the turning radius becomes about 22 m. From (d), it is seen that



(a) Turning trajectory

(b) Roll angle

(c) Steering angle

(d) Lean angle of rider's upper torso

: With arm (K_{wz}, C_{wz}), : With arm ($2K_{wz}$, $2C_{wz}$), : Without arm

Figure 3. Simulation results of steady-state turning with the effect of the rider's arm in the lean-with posture (Velocity: 35 km/h, Steering torque: -9 Nm)

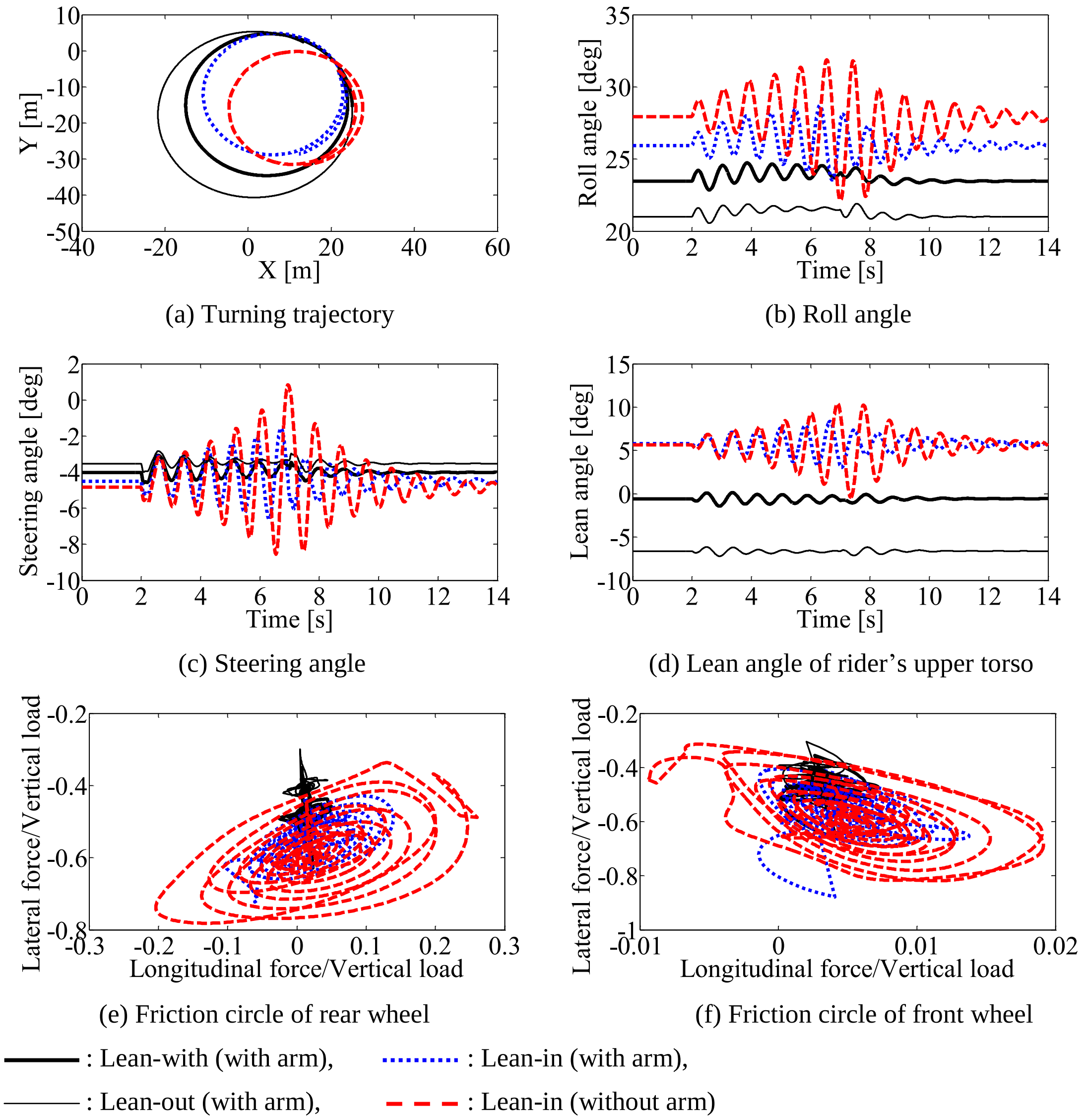


Figure 4. Simulation results of the steady-state turning with the rider's postures at 35 km/h (0 s - 2 s: μ=0.8, 2 s - 7 s: μ=0.6, 7 s - 14 s: μ=0.8)

the lean angles are within the range from -0.3 deg to -0.9 deg, and the rider's postures are regarded as lean-with. From Figure 3, it is seen that in the lean-with posture, the rider's arm stiffness affects the amplitude of the steering angle and thus the roll angle. Especially, the high stiffness of the rider's arm can obtain the large radius of turning.

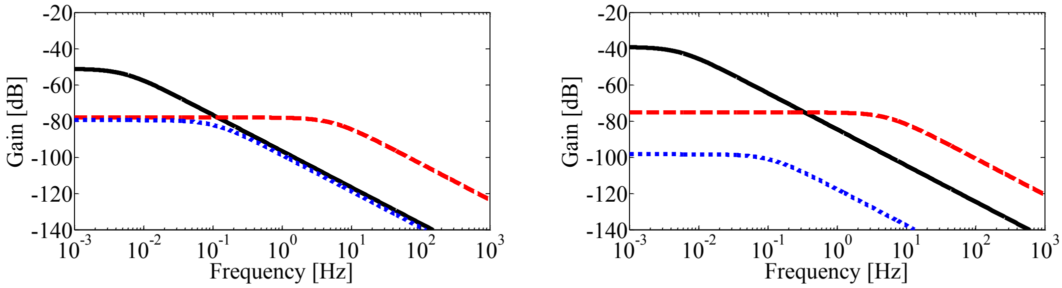
Secondly, we analyze the motorcycle motion taking account of not only the rider's arm but also the posture of his/her upper torso. We make the rider keep the postures of lean-with, lean-in and lean-out by adding the lean torque to the rider's upper torso. Figure 4 shows the simulation results of the steady-state turning with the rider's effects at 35 km/h. The friction coefficient of the road surface is originally 0.8 and suddenly decreases to 0.6 from 2 s to 7 s. (a), (b), (c), (d), (e) and (f) show the turning trajectory, the roll angle, the steering angle, the lean angle of the rider's upper torso, the friction circle of the rear wheel, and the friction circle of the front wheel, respectively.

In (b), (c), and (d), the roll angle, the steering angle, and the lean angle of the rider's upper torso generate the vibrations under the low friction road condition from 2 s to 7 s. In the case of the model with the rider's arm and the rider's posture of lean-out, because the roll angle is small at the steady-state, the vibrations are also small. In the case of the model with the rider's arm and the rider's posture of lean-in, because the roll angle is large at the steady-state, the vibrations are also large. In the case of the model without the rider's arm, this tendency further increases. In the case of the model without the rider's arm and the rider's posture of lean-in, the lateral force/vertical load of the front and rear wheels become about 0.8 in (e) and (f). Namely, the lateral forces of the front and rear wheels nearly reach the limits of the tire forces. The posture of lean-out is most stable in the steady-state turning with the same constant steering torque.

3.2 Mode analysis on the basis of the linearized steady-state turning model

We performed the mode separation and the frequency response analysis of the linearized steady-state turning model [10]. Figures 5 and 6 respectively show the frequency responses of the non-vibration and vibration modes from the steering torque to the roll angle and the steering angle. In Figures 5, α_1 and α_2 with high gain are the capsize modes. In Figures 6, β_1 with the natural frequency of about 1 Hz is the weave mode. β_2 with the natural frequency of about 6 Hz is the wobble mode. β_3 with the natural frequency of 0.8 Hz is regarded as the rider's upper torso mode. Since the rider's upper torso mode β_3 has the higher gain, it greatly affects the roll angle and the steering angle of the motorcycle.

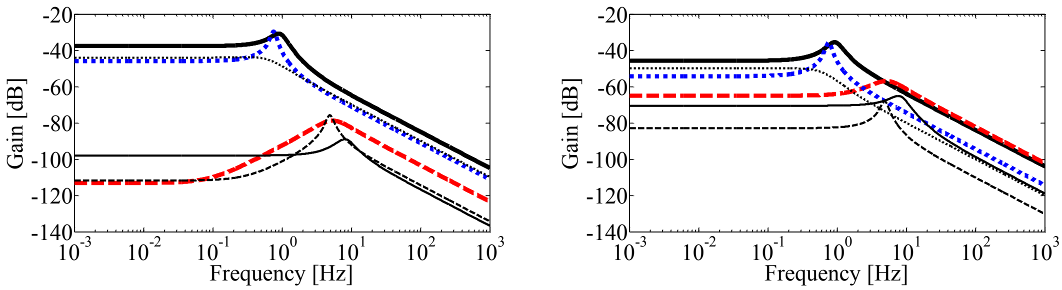
In order to analyze the motorcycle stability under the rider's postures of lean-with, lean-in, and lean-out, we perform the eigenvalue analysis of the linearized model. The real parts and the imaginary parts of the modes with the postures of lean-with, lean-in, and lean-out are plotted in Figure 7. The calculations of Figure 7 correspond to the conditions of the Figures 5 and 6. In addition, Figure 7 shows the same cases as Figure 4. The real parts of the roll capsize modes are nearly 0 from -0.045 to -0.025 in Figure 7 (a). In the case of the model with the rider's arm and the rider's posture of lean-out, the steering capsize mode, and the wobble mode are most stable. The results of Figure 7 are well in the agreement with those of Figure 4. From (c), it is seen that the weave modes are unstable in the steady-state turning with the constant steering torque of -9 Nm.



(a) Roll angle / Steering torque (b) Steering angle / Steering torque

: Roll capsize mode α_1, : Steering capsize mode α_2, : Mode α_3

Figure 5. Frequency responses of non-vibration modes in steady-state turning
(Velocity: 35 km/h, Rider's posture: lean-with)



(a) Roll angle / Steering torque (b) Steering angle / Steering torque

: Weave mode β_1, : Wobble mode β_2, : Upper torso mode β_3,
: Mode β_4, : Mode β_5, : Mode β_6,

Figure 6. Frequency responses of vibration modes in steady-state turning
(Velocity: 35 km/h, Rider's posture: lean-with)

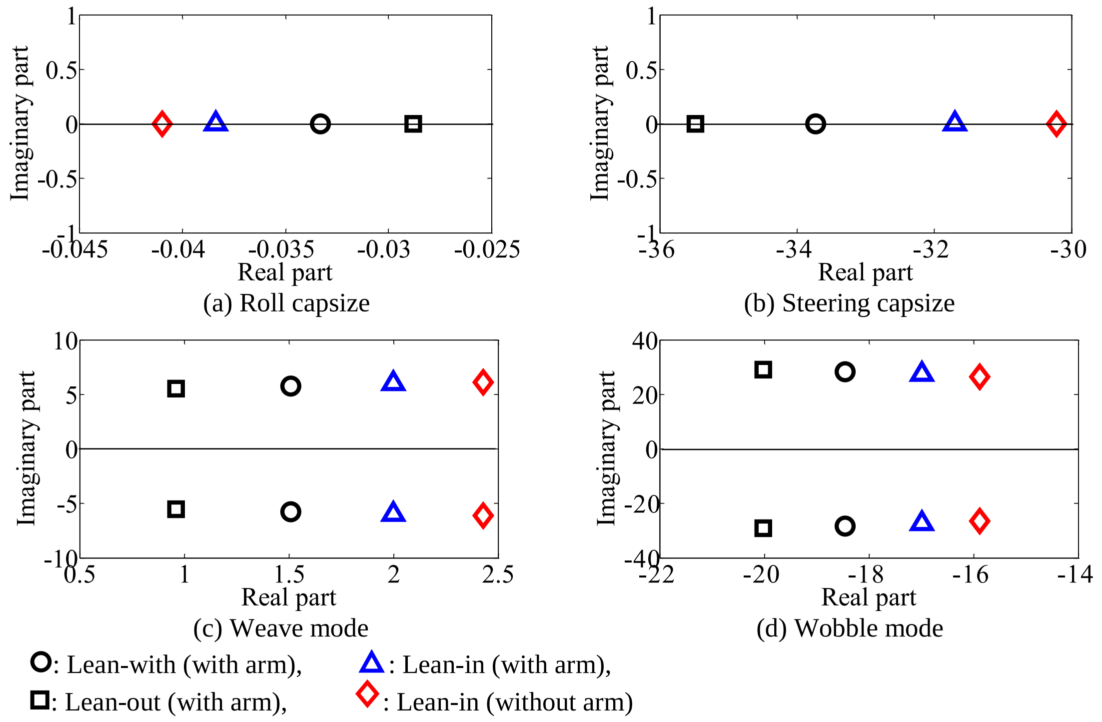


Figure 7. Eigenvalues of the linearized steady-state turning model

4 CONCLUSIONS

In this paper, we performed the motion analysis of the rider-motorcycle system taking account of the rider's effects in the steady-state turning. The results are summarized as follows.

1) We developed the model of the rider-motorcycle system by taking account of not only the leaning motion of the rider's upper torso but also the rider's arm. The nonlinear state-space model and the linearized steady-state turning model were derived.

2) For the derived nonlinear state-space model, we analyzed the effects of the rider's arm and his/her postures on the motorcycle motion by the simulations. The rider's arm stiffness affects the amplitude of the steering angle, and as a result, the roll angle and the turning radius change in the steady-state turning. The influences of the postures of lean-with, lean-in, and lean-out on the motorcycle stability are difference. It was seen that the posture of lean-out is most stable among the rider's postures in the steady-state turning with the same constant steering torque.

3) The modal analysis of the linearized model is performed. It was quantitatively shown from the frequency response analysis and the eigenvalue analysis that the posture of the lean-out is the most stable in the steady-state turning with the same constant steering torque.

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