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The Application of Handling Qualities to Bicycle Design

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ABSTRACT

Provides insight as to how bicycle/motorcycle geometry affects vehicle responsiveness. The important factors are trail and the moment of inertia of the wheels

Keywords: Single track: Design, Handling, Control, Responsiveness.

1 OVERVIEW

The application of aircraft style handling qualities to bicycles and motorcycles proceeded in discrete steps. I needed to determine that the rider changes roll rate as the first step in controlling the bike. Then I found the reaction of roll rate to handlebar displacement. The findings demanded that I study the force needed to move the handlebars. The next step was to generate a function to describe how the rider conveys his intention to the bike. I then used the force/displacement and the intention function to predict the responsiveness of the bike. An additional force that was also found to be important is “fork flop”. Finally, I used the knowledge gained to give the designer guidance as to how to build a nice handling machine.

Symbols

A	wheelbase
B	horizontal position of center of gravity measured from rear wheel contact point
h	height of the center of gravity
Kx	Longitudinal radius of gyration through the c.g.
M	mass of bike and rider
β	complement of the head tube angle
δ	Steering angle.
Nf	Normal force at the front wheel Wt B/A
ΔQ	Change in control torque
R	Front wheel radius
Rh	Radius of the handlebars
S	Fork offset
T	Trail $(R \sin(\beta)-S)/\cos(\beta)$
Wt	Weight of the bike and rider.
•	Multiplication

2 ROLL RATE

Aircraft designers do everything they can to prevent "cross control". Pilots control yaw with the feet, roll with side stick and pitch the fore/aft stick. Some aircraft have a propensity to cross control, and a touch of the pedals can cause roll in some light aircraft. The F-100 had a bad case of side stick causing yaw [1]. Bikes have only the handlebars to control both roll and yaw thus cross-control is built into the system. What do riders really do when they are controlling their machines? Direct control of direction is out of the question. Try riding and then point the front wheel in the direction you want to go; you will end up falling. In reality a rider actually turns left to go right. As the handlebar is turned left, the frame rolls to the right, then the rider can turn back to the right to stop the roll rate. The rider just makes sure he is going in the proper direction when he stops the roll. The equation for roll rate versus handlebar displacement is:

$$d(Wx)/(Rh\ d(\delta)) = (B/h)(V/A)\cos(\beta)/Rh \tag{1}$$

This is a very unsatisfactory conclusion. Having velocity in the numerator means that the bike becomes more responsive as the speed increases, but we know that is not the case. Why? The reason is the handlebars become harder to turn as the bike goes faster. So Control Force must be included in the analysis.

3 CONTROL FORCE

Two wheeled vehicles exhibit various control forces. Our main interest is the tendency of the force to change with velocity and to oppose the displacement of the handlebars. I call this the 'control spring'. The control spring is felt as a torque transmitted up the steer tube. The normal equation for a torsional spring is:

$$\Delta Q = -K\,\Delta\delta \tag{2}$$

The change in spring torque is supposed to oppose any angular deflection. However, the spring constant for a bicycle changes with velocity. So that

$$\Delta Q = (K1-(K2 * V^2))\,\Delta\delta \tag{3}$$

The bicycle effective spring constant is $(K1-K2 * V^2)$. (4)

A bicycle suffers from the strange condition of having a positive spring at low speed. Instead of opposing your hands at low speed, the handlebars tend to continue in the direction of the turn.

$$K1 = Nf * T * \cos(\beta) * \sin(\beta) \tag{5}$$

The designer may need to solve problems with low speed control. If this is the case, $\mathrm{Sin}(\beta)$ can be minimized by making the head tube as vertical as practical, thereby reducing K1.

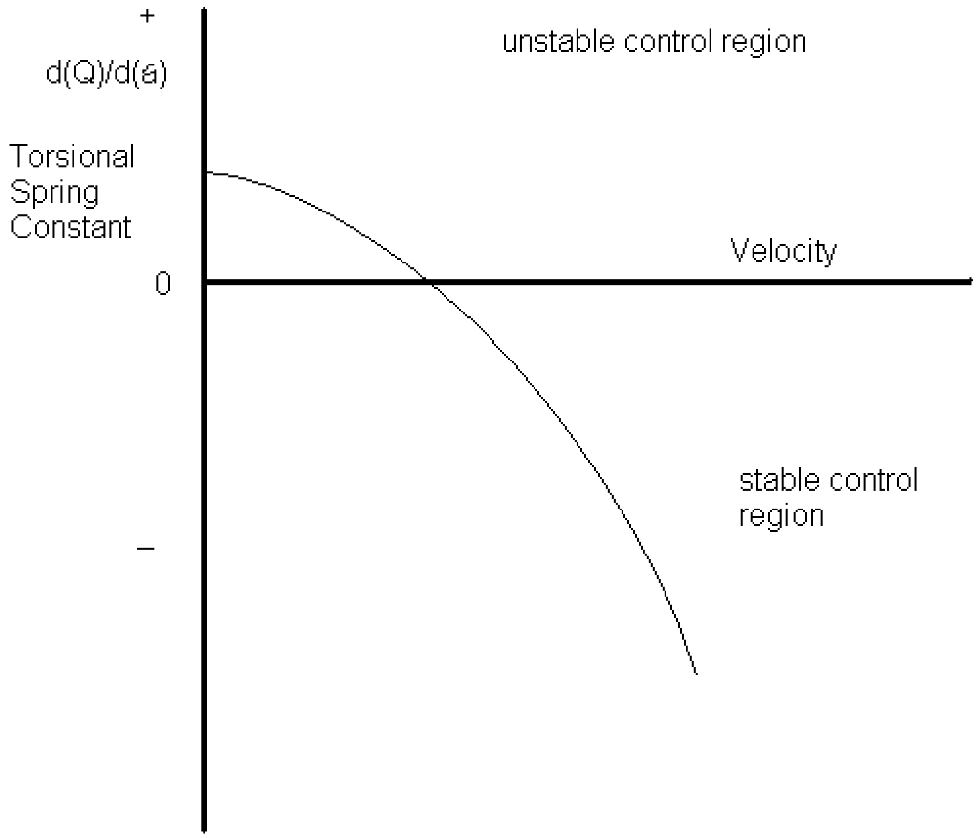


Figure 1. Spring rate vs speed

4 INTENTION FUNCTION

Once I understood the control spring, I had to figure out how force and displacement could be combined to quantify the rider's intention. I needed a function that would work for a force by itself or a displacement by itself. The only solution was to have displacement plus force. Of course they must have the same dimensions. This could only happen if the force is divided by a spring constant.

5 RESPONSIVENESS

During the development of the handling qualities theory, the authors built a variable geometry bicycle. When the bike was adjusted to be just on the edge of sensitivity, we found that we would over-control at an intermediate speed. The bike would then settle down as we went faster. This phenomenon exactly matched the predicted curve. The observed speeds of over control allowed us to estimate the spring constant for the intention function. I used several bikes and riders to confirm our findings. This allowed me to determine that the spring constant that fit the subjective feel of many people is 1500 newtons per meter. The mystery spring constant worked as a useable bike design tool for several years. We have since discovered that the USAF uses an optimal spring constant for the F-15 fighter of 7.5 lb/inch, which is approximately 1400 newtons/meter [2]. It was a nice confirmation for our subjective use of 1500 newtons/meter.

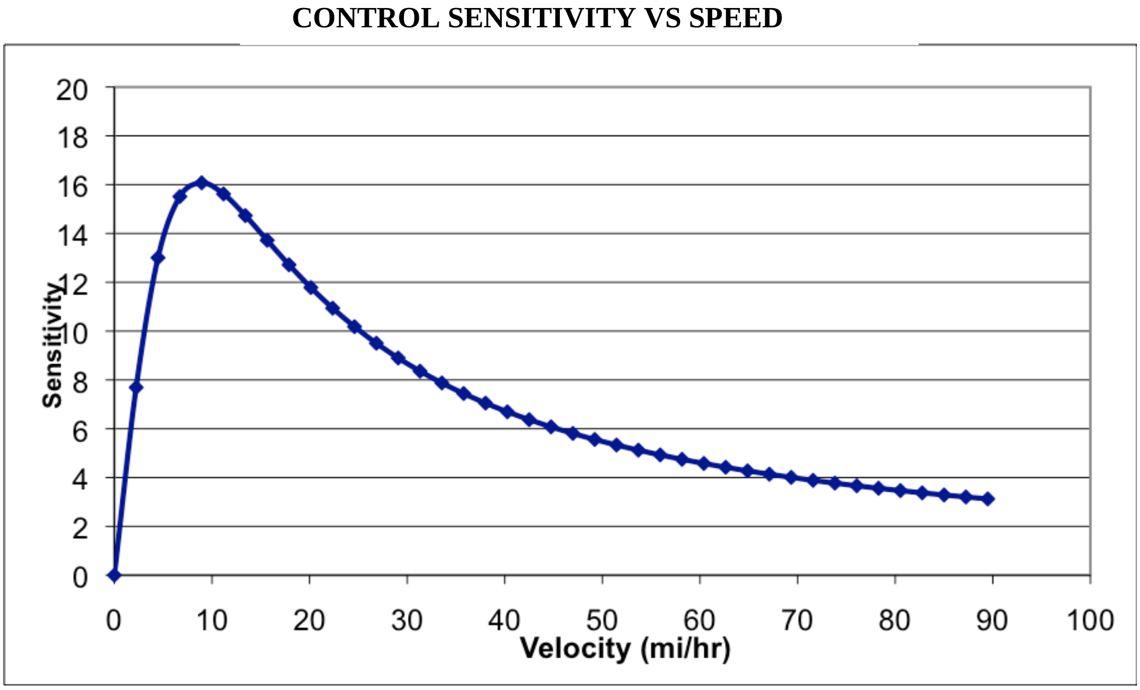


Figure 2. Vehicle responsiveness versus speed

6 MINIMUM TRAIL

All of the analysis in the world does no good if it doesn't give the designer specific advice as to how to set up the geometry of his machine. Trail and the moment of inertia of the wheels are large contributors to the magnitude of the control spring. Since bicycles have very light wheels, the mass of the wheels can be neglected by the designer [3]. The following equations do not consider the mass moment of inertia of the wheels and cannot be used to help design motorcycles.

Trail provides the feedback torque that mitigates the "over-control" condition found in too many bikes today [4]. The position of the center of gravity and the longitudinal radius of gyration through the center of gravity determine the minimum trail necessary to control sensitivity. Variable geometry bikes have been built and tested that prove the importance of trail for proper handling [5].

$$\mathrm{Tmin} = \mathrm{K5} * (B/M)(1/h^2 + 1/Kx^2) \qquad (6)$$

Note: K5 is applicable for mass measured in Kilograms. B, h, and Kx are measured in Meters. K5 has the value of 1.2 kg m^2 for light steering up to 2.4 for heavier, more normal steering.

7 MAXIMUM TRAIL

Force analysis and experimentation made it clear that another force is important to the rider. When the frame rolls from side to side a force is generated that tends to turn the handlebars into the turn. This is normally called "fork flop". Small values of fork flop are welcome. It allows the rider to keep the bike upright with very little attention. As the bike leans, the front wheel automatically turns to keep the bike upright. A significant percentage of riders have great trouble riding bikes without enough fork flop. Conversely bikes with too much trail will have problems with fork flop since the fork will turn too forcibly in the direction of frame tilt. Fork flop ranges from 75 to 200 n/rad for a nice handling bike. A maximum of something less that 300 n/rad seems to be fine. Then let

Flop = 275 newtons/radian

$$\mathrm{Tmax} = \mathrm{Flop} * \mathrm{Rh}/(\mathrm{Nf} * \cos(\beta)) \qquad (7)$$

8 APPLICATIONS

The application of Professor Patterson's bicycle handling qualities theory to fully faired Human Powered Vehicles ("HPVs") has been ongoing since 1997. The California Polytechnic State University, San Luis Obispo ("Cal Poly") student section of the American Society of Mechanical Engineers ("ASME") has sponsored a Human Powered Vehicle Team since the late 1970s. The club has designed and built fully-faired HPVs for the ASME HPV competition since its' beginning in 1983. The ASME HPV competition requires that a bicycle be able to handle high speed sprints and an approximately 40 mile (64.37 km) road-race course with many turns and with multiple rider changes.

As an amateur HPV designer/builder and advisor to the Cal Poly HPV Team since the early 1980s I have helped to build, or designed and built on my own, over 36 fully faired HPVs. For many years, most of them had mild or severe control issues over the entire range of speeds.

Two-wheeled fully-faired recumbents are unique in that the ability to control the bike by leaning is quite limited due to the low center of gravity and narrow rider compartment. The rider often has to "fly" the bike using the horizon line to maintain the wheels under the rider to keep the bike upright.

For the 1997 HPV competition, the Cal Poly HPV Team decided to design a bike based on Professor Patterson's equations. After many iterations, the computer model they chose forecast stability at both low and high speeds for an HPV with a short wheelbase and under-seat steering. Historically, such designs have often been unstable at low speed. The frame built using the model geometry was very stable at low speed, but due to fairing issues was not tested at speeds over 35 mph (56.33 kph).

Over the years as Professor Patterson refined his theory the Cal Poly HPV Team continually refined their bike geometry for a variety of designs: short and long wheelbase, under-seat and over-seat steering, etc. Each year the bike handling characteristics were comparable to the computer model chosen, as long as the as-built bicycle conformed to the model parameters (not always the case!). The result has been a series of Cal Poly HPV Team bikes that are easy to launch, can accommodate a wide range of riders and rider capabilities, and have excellent control at high speed, culminating in a 59.89 mph by student Ron Layman in *Velox Solium* at Battle Mountain, Nevada in 2004.

An unofficial, subjective comparison of the controllability of student-built bikes at high speeds has been conducted for years at the end of the ASME sprint course. Students and advisors waiting there for the HPVs to finish their high-speed run would always know when Cal Poly was coming because it was the only fully-faired bike that did not wobble through the speed traps, but instead rode straight through the course.

Patterson Control Model

Input Cells
Calculated Cells

Inputs

a	1.092	m	Wheelbase
b	0.508	m	Horizontal: rear axle to CG
h	0.508	m	Height of CG
kx	0.41	m	Radius of Gyration [seat vertical=.44m, 60 deg.=.41m, 45 deg.=.35m, 30 deg. Laid back=.31m]
beta	14	deg	Head Tube Angle from vertical
s	-0.044	m	Fork Offset (minus?)
Rt	0.24	m	Rt in this code is the front wheel radius from the axle to the ground.
m	92	kg	Mass of Rider & Bike
Rh	0.145	m	Handlebar Radius

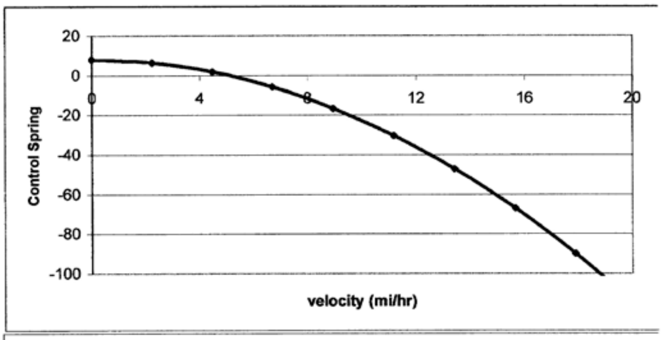
Front Weight X Wheelbase / Total Mass = Distance Rear Axle to C.G.

Velox Solium Plot as-built

beta	0.244346	rad
mg	902.52	N
T=	**0.105186**	**Trail (m)**
k1=	7.941343	
k2=	1.530954	
k3=	0.000667	
k4=	0.888549	

FF= **295.52 Fork Flop-Best Below 275**

v (m/s)	v (mi/hr)	spring	sens
0	0	7.941343	0
1	2.237	6.410388	7.691272
2	4.474	1.817525	13.00536
3	6.711	-5.83725	15.51257
4	8.948	-16.5539	16.07434
5	**11.185**	**-30.3325**	**15.61818**
6	13.422	-47.173	14.73192
7	15.659	-67.0754	13.71845
8	17.896	-90.0397	12.71682
9	**20.133**	**-116.066**	**11.78384**
10	22.37	-145.154	10.93767
11	24.607	-177.304	10.17926
12	26.844	-212.516	9.502476
13	29.081	-250.79	8.898791
14	**31.318**	**-292.126**	**8.359406**
15	33.555	-336.523	7.87612
16	35.792	-383.983	7.441634
17	38.029	-434.504	7.049603
18	**40.266**	**-488.088**	**6.694573**
19	42.503	-544.733	6.371883
20	44.74	-604.44	6.077552
21	46.977	-667.21	5.80818
22	49.214	-733.041	5.560855
23	**51.451**	**-801.934**	**5.33308**
24	53.688	-873.888	5.122702
25	55.925	-948.905	4.927862
26	58.162	-1026.98	4.746948
27	**60.399**	**-1108.12**	**4.578556**
28	62.636	-1192.33	4.421458
29	64.873	-1279.59	4.274579
30	67.11	-1369.92	4.136974
31	69.347	-1463.31	4.007807
32	**71.584**	**-1559.76**	**3.886337**
33	73.821	-1659.27	3.771908
34	76.058	-1761.84	3.663933
35	78.295	-1867.48	3.561889
36	**80.532**	**-1976.18**	**3.465306**
37	82.769	-2087.94	3.373764
38	85.006	-2202.76	3.286881
39	87.243	-2320.64	3.204315
40	**89.48**	**-2441.59**	**3.125755**



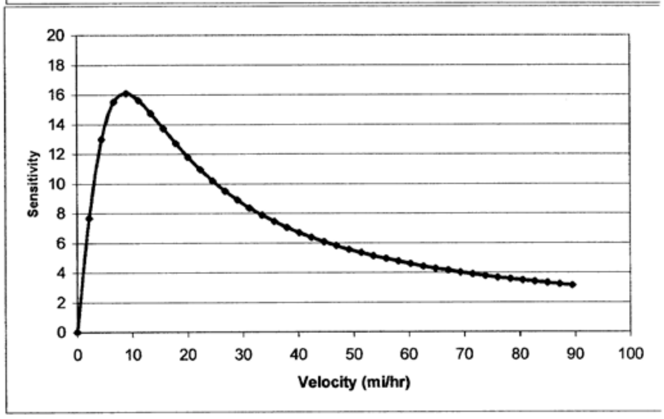


Figure 3. *Velox Solium* graph, 2004

Figure 4. *Velox Solium* Frame 2004

After observing the consistent stability results for the Cal Poly bikes, when I started to design my latest HPV series, *Primal*, I used Professor Patterson's equations in order to maximize stability at both low speed (launch and first 500 meter stability) and high speed (70 + mph [112+ kph]). After inputting various values such as head tube angle, trail and wheelbase, I started iterating hundreds of Excel computer graphs, trying to find the one that would give me the stability values I was looking for. The chosen model (Figure 4) provided a compact wheelbase that predicted good stability at both low and high speed.

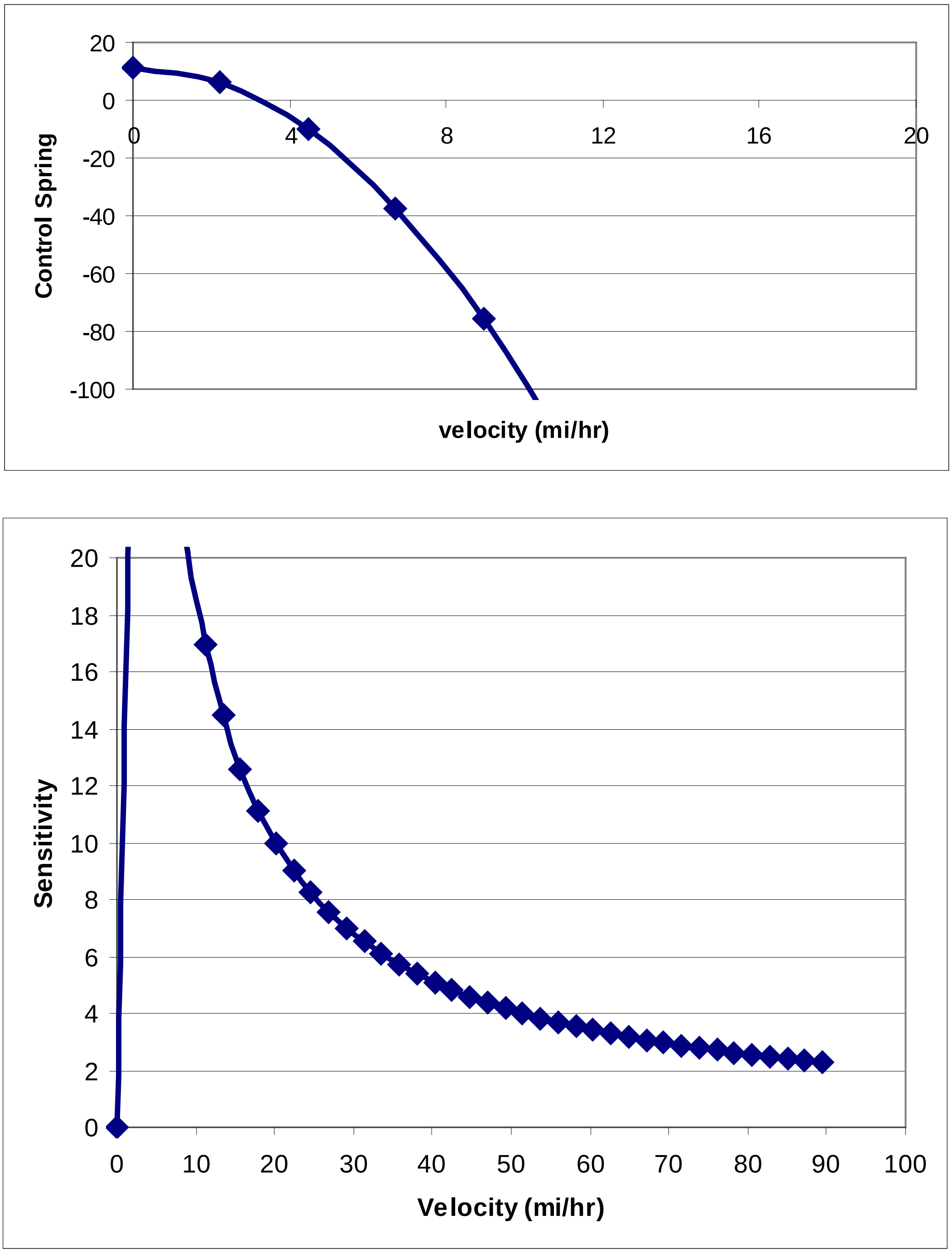


Figure 5. Graph of *Primal 1* 2008

We built the prototype bike, *Primal 1* over the summer of 2008, with the frame matching specific dimensions based on the computer model. It was first ridden at the World Human Powered Speed Challenge (WHPSC) in Battle Mountain Nevada. This competition is a series of straight-line sprints over six days for the fastest human powered vehicles in the world. The course is six miles (9.66 kph) in length, with a five mile (8.05 kph) length to get up to maximum speed, 200 meter timing traps and one mile (1.609 kph) to slow down. Normally a bike/rider must be able to reach a qualifying speed of more than 45 mph (72.42 kph) to compete.

Figure 6. *Primal 1* frame 2008

The rider of *Primal 1*, Ron Layman is a former Cal Poly HPV Team rider with years of experience riding fully-faired recumbents. After riding the bike all week, his conclusion was that it was very stable at low speeds and rock-solid stable at over 62 mph (99.78 kph), the highest speed he attained all week. Both the rider and builder/designer felt that the frame was capable of higher speeds but was limited by the fairing design.

In 2009, we determined that the bike frame had to have a much lower center of gravity and a few other changes, but we did not want to affect the bike's basic handling qualities. We input various new values of wheelbase and head tube angle into the Patterson equations until we produced an Excel graph that was comparable to the 2008 graph. During the summer of 2009 a frame was built to the new geometry specifications, but would the handling remain the same?

We tested *Primal 2* at the 2009 WHPSC. The new configuration had virtually the same handling characteristics as *Primal 1*, proving that the equivalent curves produced by use of the Patterson equations accurately predicted the equivalent controllability when the bike parameters changed. With a new frame and a new, more streamlined fairing, *Primal 2* reached 66.595 mph (107.175 kph) in 2009.

For 2010 we used the 2009 frame with no changes, but improved the aerodynamic properties of the fairing. The result was an increase of another 4 mph (6.44 kph) for a top speed of 70.40 mph (113.30 kph). This puts *Primal 2* and Ron Layman in the top ten fastest HPVs in history after only 3 years of competition.

Figure 7. *Primal 2* Frame, 2009/2010

9 CONCLUSIONS

The handling characteristics of “standard” and unusual geometry bicycles can be predicted by use of the Patterson equations. Similar handling feel can be developed from differing geometries by iterating the parameters so that the curves generated are equivalent. These equations give the designer the equivalent of the adjustment used to change the sensitivity of a computer mouse.

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