

Simulation and Control of the *Anaconda*

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ABSTRACT

In this paper, a new single-track vehicle, the *Anaconda*, is presented and modelled according to a multibody theory. This articulated vehicle begins with a traditional bicycle, called the *head module*, followed by a succession of so-called *pedal modules*, each module being connected to the preceding one by a spherical joint. To assess its dynamic behaviour, the model of an *Anaconda* with two *Pedal Modules* is simulated under the EASYDyn framework, a multibody library based on the minimal coordinates approach. The simulation of such a vehicle cannot be performed without the riders action, consisting in torques applied on the handlebars. The latter is implemented by controllers designed by optimal control, from the out-of-plane dynamics of the vehicle going straight ahead at 20 km/h. Firstly, two optimal controllers are determined separately for the *Head Module* alone on one hand, and for the *Pedal Module* alone on the other hand. They are then implemented on the *Anaconda* and it appears that the vehicle is stable but that the handling of the pedal modules is delicate. Finally, the difficulty of riding the *Anaconda* is evaluated through the so-called rideability index, which increases, as expected with the amount of *Pedal Modules*.

Keywords: multibody, bicycle dynamics, optimal control, path-following, rideability.

1 INTRODUCTION

Innovative ideas must be withstood by research in order, for instance, to build products with appropriate features. This fact lead the Faculty of Engineering of the University of Mons to get involved in the *Anaconda* project. The *Anaconda* project consists in designing an articulated in-line polycycle propelled by man power and able to follow any winding road. The *Anaconda*, shown in Figure 1, consists of a *Head Module* (HM), which resembles the traditional bicycle, followed by several *Pedal Modules* (PM) connected to each other by spherical joints.

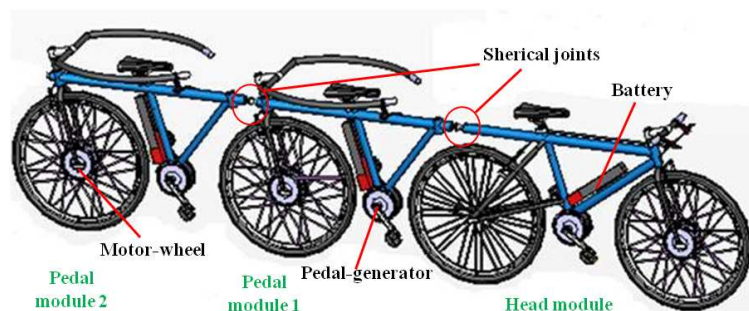


Figure 1. The *Anaconda* with 2 pedal modules

To allow a maximum number of modules, each module is equipped with a handlebar in order to control itself its equilibrium. The *Anaconda* is a man-powered vehicle: the driver (rider on the HM) and passengers (riders on PMs) act on their pedals to feed the pedal-generators that supply electrical power to the motor-wheels either directly or through batteries. Power electronic devices are foreseen to manage this task, which is, on the contrary of the balance, centralized and controlled by the driver through a dashboard. Brake system is also centralized and in case of emergency mechanical brakes can be used. The driver decides either to accelerate, to brake or to steer, and passengers must collaborate.

As designed, the *Anaconda* is a single-track vehicle and such vehicles are largely described in scientific papers in terms of modelling aspects or dynamical studies. Sharp [1, 3] built the equations of motion of a free motorcycle according to the Lagrange's approach, considering rolling wheel as force producer with a specific tire model. He studied the evolution of the eigenvalues in terms of vehicle speed and tire parameters and clearly emphasized the existence of 3 typical modes referred to as capsize, weave and wobble. A similar approach was used by Koenen [2]. Schwab et al [4]–[6] present a benchmark bicycle model for which the linearized dynamic equations are used to study its balance, the steering, or the uncontrolled motion. In [7]–[10], the same model is used for control purpose. A model must exhibit the physical aspect of the system for which it is built: so Aström et al [11] present a simplified model that helps to analyze the balancing and steering problems.

This paper focuses on the stability of the *Anaconda* and tries to demonstrate that its handling is possible. The *Anaconda* is modelled as a multibody system and simulated within the `EasyDyn` framework [12], a C++ multibody library based on the minimal coordinates approach. The model follows the classical assumptions of literature regarding the number of bicycle parts to take into the account, their elasticity, the model of the tire-road interaction and the rider model. The rider controls his module through a steering torque applied on the handlebar and consists of two parts: the upper body and the lower body connected to each other by a revolute joint which allows the upper body to rotate about the longitudinal axis with respect to the lower one. The latter is rigidly connected to the frame [8], [13]. All bodies are assumed to be rigid. Tires are considered as force producers and the model of the University of Arizona [14] is used.

First, we present the multibody model of the *Anaconda*, developed within the `EasyDyn` framework. The model is fully described in terms of geometry, inertia properties and configuration parameters. The kinematics largely relies on the homogeneous transformation matrix formalism. The non linear differential equations of motion are then derived, linearized, reduced to the out-of-plane motion and recast in the state space form. The linear quadratic regulator theory [15] is used to build a controller for the head module alone and a controller for the pedal module alone. These controllers are then implemented as is inside an *Anaconda* with 2 pedal modules. The latter is stable but the pedal modules prove to be difficult to drive. Finally, the handling of the vehicle is assessed through the so-called *rideability index* [16] which appears to increase with the number of modules.

2 THE *Anaconda* MODEL

2.1 The mechanical model

The mechanical model of the *Anaconda* with one pedal module, shown in Figure 2, consists of nine rigid bodies. The head module comprises the 5 bodies classically identified on bicycle [8]: the HM frame denoted by body 1, including the driver lower body, the front fork and handlebar

assembly (body 2), the front and rear wheels denoted by bodies 3 and 5, and the driver upper body denoted by body 4. The PM has four bodies: the PM frame with the passenger lower body attached to it, the passenger upper body, the fork and handlebar assembly and the wheel. They are denoted by body 6, 7, 8 and 9 respectively.

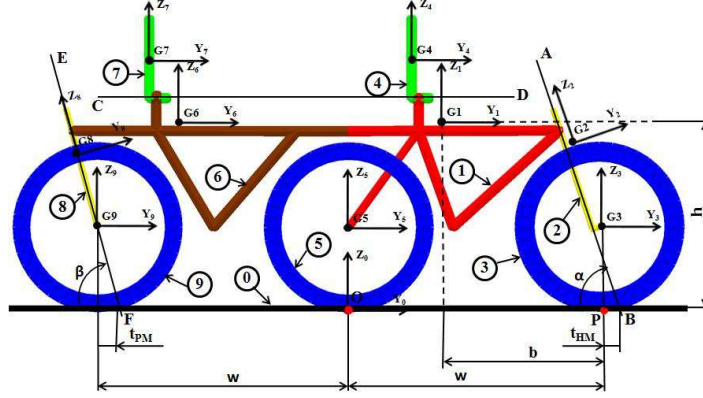


Figure 2. The rider-vehicle mechanical model

A right handed body fixed coordinate system is attached to each body at the center of gravity. Figure 2 gives a screenshot of the *Anaconda* model in an upright position in which all the YOZ planes of the local coordinate systems lie in a vertical longitudinal plane. The distribution of mass is assumed to be symmetric transverse to this plane. The inertia data are estimated from the benchmark bicycle [7, 8] and given in Table 1, with respect to the local axes.

The bodies are interconnected by mechanical joints that restrain their relative motions, each joint having its own configuration parameters (articular variables). The tire wheel-road contact is considered as force producer, so that we have an open-loop multibody system. With respect to the ground (body 0), the HM frame enjoys six degrees of freedom as a free body in 3D motion, denoted by $q[0]$ to $q[5]$. The configuration parameters $q[0]$, $q[1]$, and $q[2]$ correspond to the three displacements along OX, OY and OZ global axes respectively while $q[3]$, $q[4]$, and $q[5]$, in this order, represent the classical succession of the yaw (Z), pitch (X) and roll (Y) angles. The HM fork is connected to the HM frame by a revolute joint allowing its relative motion about the steering axis AB. This joint introduces one configuration parameter denoted by $q[6]$ and called the HM steering angle. In the same way, the HM front and rear wheels are connected to the HM fork and the HM frame respectively by a revolute joint, whose axis is perpendicular to the vehicle plane (OX axis of the local coordinate system). The corresponding rotation angles are denoted by $q[7]$ and $q[9]$. The relative motion between the driver upper body and the lower one is defined as a rotation about the hip's axis CD. The related angle is called the body lean angle and is denoted by $q[8]$.

The frame of the pedal module is attached to the frame of the head module by a spherical joint defined as a succession of relative yaw, pitch and roll angles, denoted by $q[10]$, $q[11]$ and $q[12]$. The steering angle, the wheel rotation angle and the lean angle are denoted by $q[13]$, $q[14]$ and $q[15]$ respectively. Table 2 summarizes the information about all joints (configuration parameter and connected bodies).

It is of interest to note that, although the lean angles between the upper and lower parts of the driver are present in the model, they are frozen for all cases presented in this paper.

Table 1. Parameter values of the rider-vehicle's mechanical model. G_i and G_0 are centers of gravity of bodies i and 0

Parameter	Symbol		Value	
	For the HM	For the PM	For the HM	For the PM
Wheel base	w	w	1.07 m	1.07
Trail	t_{HM}	t_{PM}	0.08 m	0.094 m
Head angle	α	β	$2\pi/5 \text{ rad}$	$5\pi/12 \text{ rad}$
Forward speed	v		20 km/h	
Body	Mass	Position of G_i w.r.t $G_0: (x, y, z)$	Mass moment of inertia: $(I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}) \text{ kgm}^2$	
1	34 kg	(0,0.345,0.765) m	(4.667,3.869,1.272,0,0,-1.3)	
2	4 kg	(0,0.95,0.7) m	(0.06,5.843e-2,7.5866e-3,... ...,0,0,-9.1192e-3)	
3	3 kg	(0,1.07,0.35) m	(0.28,0.1405,0.1405,0,0,0)	
4	51 kg	(0,0.27,0.99) m	(5.186,4.299,1.413,0,0,-1.444)	
5	3 kg	(0,0,0.35) m	(0.28,0.1405,0.1405,0,0,0)	
6	34 kg	(0,-0.775,0.765) m	(4.667,3.869,1.272,0,0,-1.3)	
7	51 kg	(0,-0.85,0.99) m	(5.186,4.299,1.413,0,0,-1.444)	
8	4 kg	(0,-1.161,0.69) m	(0.06,5.9227e-2,6.7726e-3,... ...,0,0,-6.4128e-3)	
9	3 kg	(0,-1.07,0.35) m	(0.28,0.1405,0.1405,0,0,0)	
Road-tire contact point P coordinates with respect to G_1			$(0, b, -h): (0,0.725,-0.765) \text{ m}$	
HM and PM hip's position w.r.t $G_0: (0,0.3,0.9) \text{ m}$ and $(0,0.82,0.9) \text{ m}$				

The presented model can be easily extended to an *Anaconda* with several pedal modules. In a general way, for n pedal modules, the mechanical model has $n_b = 6 + 4 * n$ rigid bodies (ground included) and $n_{cp} = 10 + 6 * n$ relative degrees of freedom (configuration parameters). Especially, for an *Anaconda* with two pedal modules, the vehicle under study in this paper, we have 14 rigid bodies and 22 relative degrees of freedom. It is clear that the n_{cp} configuration parameters gathered in the vector \mathbf{q} are independent and constitute a set of minimal coordinates.

2.2 Description of kinematics

To describe the spatial situation of each rigid body with respect to a reference one, we use the homogeneous transformation matrix formalism (HTM) [17]. The HTM, $T_{i,j}$, is a mathematical tool used to express the relative situation of body fixed frame j with respect to the body fixed frame i . This is a 4x4 matrix which can be partitioned in the following way:

$$\mathbf{T}_{i,j} = \begin{pmatrix} \mathbf{R}_{i,j} & \{\mathbf{r}_{j/i}\}_i \\ 000 & 1 \end{pmatrix} \quad (1)$$

where i and j are frames attached to the centers of gravity of body i and j respectively, $\{\mathbf{r}_{j/i}\}_i$ is the coordinate vector of frame j with respect to frame i projected in the reference frame i and $\mathbf{R}_{i,j}$ is the rotation tensor describing the orientation of frame j with respect to frame i .

Table 2. Bodies et their relative degrees of freedom

Body		Joint			relative conf. param	
No.	name	name	w.r.t	n_{rdof}	denoted	name
0	ground	fixed	ground	0	-	-
1	HM frame	free	ground	6	q[0]	-
					q[1]	-
					q[2]	-
					q[3]	HM yaw angle
					q[4]	HM pitch angle
					q[5]	HM roll angle
2	HM fork	revolute	HM frame	1	q[6]	HM steering angle
3	HM front wheel	revolute	HM fork	1	q[7]	-
4	HM rider	revolute	HM frame	1	q[8]	driver body lean angle
5	HM rear wheel	revolute	HM frame	1	q[9]	-
6	PM frame	spherical	HM frame	3	q[10]	PM relative yaw angle
					q[11]	PM relative pitch angle
					q[12]	PM relative roll angle
7	PM rider	revolute	PM frame	1	q[13]	passenger body lean angle
8	PM fork	revolute	PM frame	1	q[14]	PM steering angle
9	PM rear wheel	revolute	PM fork	1	q[15]	-

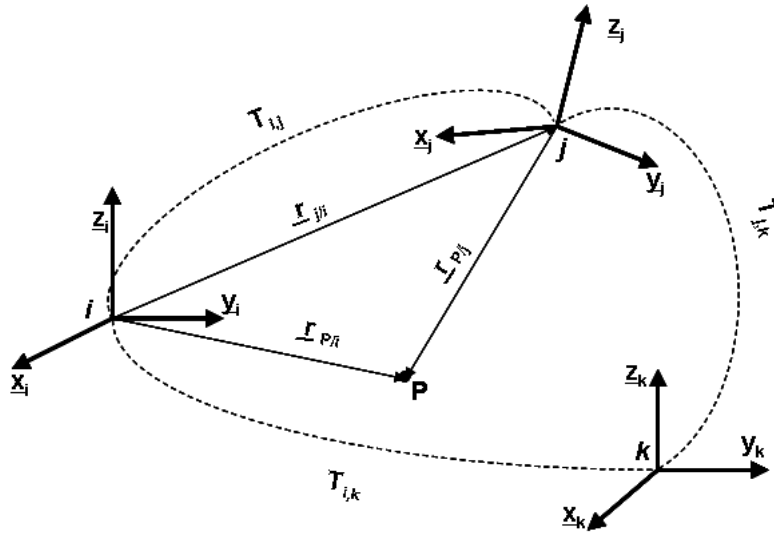


Figure 3. Coordinate vector and HTM

The HTM enjoy the following property

$$\mathbf{T}_{i,j} = \mathbf{T}_{i,k} \cdot \mathbf{T}_{k,j} \quad (2)$$

which allows to compose a complex HTM from the successive multiplication of simple ones.

The HTM, $\mathbf{T}_{0,i}(\mathbf{q})$, of any frame i with respect to the reference frame 0 expressed in terms of the vector \mathbf{q} (the n_{cp} configuration parameters) contains all the informations to derive the kinematics

of the *Anaconda*, viz the position, the velocity and the acceleration of each body i . A complete development can be found in [17]. The main result is that the translational and rotational velocities $\underline{\mathbf{v}}_i$ and $\underline{\boldsymbol{\omega}}_i$ of each body i depend linearly on the time derivatives of the configuration parameters

$$\{\underline{\mathbf{v}}_i\}_0 = \{\mathbf{J}_{Si}\}_0 \cdot \underline{\dot{\mathbf{q}}} \quad (3)$$

$$\{\underline{\boldsymbol{\omega}}_i\}_0 = \{\mathbf{J}_{\omega i}\}_0 \cdot \underline{\dot{\mathbf{q}}} \quad (4)$$

where \mathbf{J}_{Si} and $\mathbf{J}_{\omega i}$ are called velocity transformation matrices of $(3 \times n_{cp})$ dimension.

The translational and rotational accelerations $\underline{\mathbf{a}}_i$ and $\underline{\dot{\boldsymbol{\omega}}}_i$ of each body i are given by

$$\{\underline{\mathbf{a}}_i\}_0 = \frac{d}{dt}\{\underline{\mathbf{v}}_i\}_0 = \{\mathbf{J}_{Si}\}_0 \cdot \underline{\ddot{\mathbf{q}}} + \{\dot{\mathbf{J}}_{Si}\}_0 \cdot \underline{\dot{\mathbf{q}}} \quad (5)$$

$$\{\underline{\dot{\boldsymbol{\omega}}}_i\}_0 = \frac{d}{dt}\{\underline{\boldsymbol{\omega}}_i\}_0 = \{\mathbf{J}_{\omega i}\}_0 \cdot \underline{\ddot{\mathbf{q}}} + \{\dot{\mathbf{J}}_{\omega i}\}_0 \cdot \underline{\dot{\mathbf{q}}} \quad (6)$$

2.3 Applied forces

The applied forces considered in the model are the gravity, the steering torques on handlebars and the tire forces. The latter are determined according to the model of the University of Arizona [14] which involves any combination of longitudinal slip, lateral slip and camber. The ground surface is assumed to be flat. The tire parameters are summarized in table 3.

Table 3. Tire model's parameter values

Symbol	Parameter	Value
r_1	Outer radius	0.35 m
r_2	Equivalent torus radius	0.03 m
K_Z	Vertical stiffness	170000 N/m
C_Z	Vertical damping	142.828 Ns/m
F_{znom}	Nominal vertical force on tire	computed for all tires in the nominal position: the vehicle in the vertical position
$C_{longnom}$	Nominal longitudinal stiffness	3335 N/rad
C_{latnom}	Nominal cornering stiffness	3335 N/rad
$C_{cambernom}$	Nominal camber stiffness	490 N/rad
f_{Clbs}	Static Coulomb friction coefficient	1
f_{Clbd}	Dynamic Coulomb friction coefficient	0.8
n_{long}, n_{lat} and n_{camber}	exponent of the stiffness-normal force dependency	0.1

All aerodynamic effects, driving torques on pedals and upper body's lean torques are neglected.

2.4 Equations of motion

From the kinematics and the applied forces, the equations of motion can be derived by application of the principle of virtual powers [12]. As the configuration parameters are independent, the n_{cp} equations of motion are ordinary differential equations (ODE) of the form

$$\mathbf{M}(\underline{\mathbf{q}}) \cdot \underline{\ddot{\mathbf{q}}} + \underline{\mathbf{h}}(\underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}) = \underline{\mathbf{g}}(\underline{\mathbf{q}}, \underline{\dot{\mathbf{q}}}, t) \quad (7)$$

with \mathbf{M} the mass matrix of dimension $(n_{cp} \times n_{cp})$, \mathbf{h} of dimension $(n_{cp} \times 1)$, gathering the centrifugal and Coriolis contributions and \mathbf{g} of dimension $(n_{cp} \times 1)$, gathering the external forces contributions.

These equations are integrated numerically in their second-order form by the help of the Newmark integration formulas with $\beta = 0.25$ and $\gamma = 0.5$.

2.5 The EasyDyn framework

The dynamic computational tasks of this study are performed under the EasyDyn framework [12]. EasyDyn is not a software but an open source C++ library coupled with a symbolic tool called CAGeM (Computer-Aided Generation of Motion).

Simulating a mechanical system is completed by the following succession of tasks (more details can be found in [12] or on the EasyDyn web site)

- The description of the kinematics in terms of homogeneous transformation matrices. This is performed in the form of a script (initially MuPAD, recently ported to Xcas) which also defines the inertia properties, the initial conditions and some flags¹.
- The treatment of the script with CAGeM which symbolically computes expressions of velocities and accelerations by derivation of the homogeneous transformation matrices, and generates a C++ program linkable against the EasyDyn library, which implements the simulation.
- The adaptation of the created C++ file in order to define applied forces, body shapes and to adjust the simulation process (static equilibrium, linearization, computation of poles, ...).
- The compilation of the final C++ file in order to get an executable program.
- The run of the simulation program and the analysis of the results (an other open source software called EasyAnim is also provided to animate the motion of the system).

Besides the fact that it is free and completely mastered by the department, EasyDyn is particularly interesting for this application as everything can be parametrized: number of pedal modules, geometry of the module, ... Moreover, differential equations can be naturally added to the ones describing the motion of the mechanical system. This feature is used namely for the integral action of the controllers presented later.

3 APPROACH FOR EVALUATING THE RIDEABILITY

3.1 Introduction

The purpose of the study is to figure out the chances of the *Anaconda* to be driven by standard people. The stabilization of two-wheelers is generally managed in simulation by designing a controller adjusting the steering torque on the handlebar [8]–[11] and [13], [16] from the state of the vehicle. Starting from the fact that a bike can be driven, we wanted to know whether a controller could be found for the pedal module that is achievable by human being and, hopefully the same for any module. To answer this question, a controller has been established for a head module alone and for a pedal module alone. The latter are then implemented on an *Anaconda* with 2 pedal modules and the performances are analysed.

¹For the purpose of illustration, the script related to the head module is given in appendix A

3.2 Steps to build the controllers

3.2.1 Step 1

The equations of motion are linearized about a stationary state corresponding to a straight-line motion at a **constant velocity of 20 km/h**. The resulting equations are written

$$\mathbf{M}\Delta\ddot{\mathbf{q}} + \mathbf{CT}\Delta\dot{\mathbf{q}} + \mathbf{KT}\Delta\mathbf{q} = \mathbf{F}\mathbf{u} \quad (8)$$

where \mathbf{M} , \mathbf{C} and \mathbf{K} are the mass, damping and stiffness matrices respectively, \mathbf{F} the influence matrix and \mathbf{u} the input, which corresponds here to the torque exerted on the handlebar.

3.2.2 Step 2

The matrices are retrieved under Matlab, **reduced to the out-of-plane motion**, and used to build the equivalent state-space form of the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (9)$$

where the state vector \mathbf{x} contains the out-of-plane configuration parameters \mathbf{q}^* and their time derivatives $\dot{\mathbf{q}}^*$

$$\dot{\mathbf{x}} = \left\{ \begin{array}{c} \mathbf{q}^* \\ \dot{\mathbf{q}}^* \end{array} \right\} \quad (10)$$

The evolution matrix \mathbf{A} and the command matrix \mathbf{B} can be built from the reduced mass, damping, stiffness and influence matrices, as

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{*-1}\mathbf{K}^* & -\mathbf{M}^{*-1}\mathbf{C}^* \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{*-1}\mathbf{F}^* \end{bmatrix} \quad (11)$$

with $\mathbf{0}$ and \mathbf{I} are zero and identity matrices of appropriate dimensions.

The number of states, denoted by n , is equal to twice the number of out-of-plane configuration parameters, that's to say 8 for the HM and 6 for the PM.

3.2.3 Step 3

A state-space feedback controller is designed according to the scheme presented in figure 4. The system is augmented so as to introduce a new state variable which is the integral of the error between the output variable y and the target r . The presence of the integral action on the error allows to capture the properties of PID controllers [15]. The state-space form of the corresponding augmented system is written

$$\dot{\mathbf{x}}^* = \mathbf{A}^*\mathbf{x}^* + \mathbf{B}^*u + \mathbf{E}^*r \quad (12)$$

$$y = \mathbf{C}^*\mathbf{x}^* \quad (13)$$

in which \mathbf{x}^* is the augmented state vector and \mathbf{A}^* , \mathbf{B}^* , \mathbf{C}^* , \mathbf{E}^* the matrices of the augmented system, built from the original one as

$$\mathbf{x}^* = \begin{bmatrix} \mathbf{x} \\ e \end{bmatrix} \quad \mathbf{A}^* = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix} \quad \mathbf{B}^* = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \quad \mathbf{E}^* = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \quad \mathbf{C}^* = \begin{bmatrix} \mathbf{C} \\ \mathbf{0} \end{bmatrix}^T \quad (14)$$

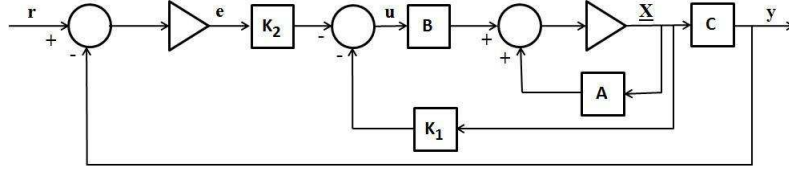


Figure 4. Closed loop state feedback with Integral action

As it is a state feedback approach, the input \underline{u} is determined as a linear combination of the state variables

$$\underline{u} = -\mathbf{K}^* \underline{x}^* \quad (15)$$

where the state feedback gain vector \mathbf{K}^* is of the form $\mathbf{K}^* = [\mathbf{K}_1 \quad \mathbf{K}_2]$, \mathbf{K}_1 being the gain vector applied on the initial system states \underline{x} and \mathbf{K}_2 the gain applied on the state created by the integral action \mathbf{e} .

This way of working assumes that all state variables are directly measurable. This hypothesis is reasonable due to the numerous sensing capabilities of the human being.

Besides the benefit of the integral action, another advantage of this scheme is that it can be designed in a systematic way by using the theory of optimal control. The method consists in minimizing the following quadratic cost function [15]

$$\tilde{\mathbf{J}} = \int_0^{\infty} (\underline{x}^T Q_x \underline{x} + \mathbf{u}^T Q_u \mathbf{u}) dt \quad (16)$$

where $Q_x \geq 0$ and $Q_u > 0$ are symmetric, positive semi-definite matrices of the appropriate dimensions. The corresponding optimal feedback gain vector is computed by the single command `lqr` under MATLAB [18].

For the method to be applicable, the system must be controllable, that's to say that the controllability matrix \mathbf{W}

$$\mathbf{W} = [\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}] \quad (17)$$

must have full rank.

In our case, the output variable y is the lateral position of the contact point of the wheel with the ground and it is compared with the prescribed trajectory r . In practice, it is approximated by the lateral coordinate of a point P attached to the bicycle frame. For example, for the head module, the coordinates of this point in the coordinate system of the main frame are $(0, b, -h)$, with b and h depicted in Figure 2. For designing the controller, the output must be expressed linearly in terms of the state variables

$$\underline{y} = \mathbf{C}\underline{x} \quad (18)$$

which gives, for the head module

$$y = q[1] - b * q[3] - h * q[4] \quad \text{or better} \quad (19)$$

$$= [1 \quad -b \quad -h \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \underline{x} \quad (20)$$

3.2.4 Step 4

The established controller is implemented in the complete multibody model, in order to estimate its robustness with respect to the nonlinearities and the eventual coupling with the in-plane motion.

4 SIMULATION RESULTS

4.1 The head module

The multibody model of the head module involves 10 degrees of freedom and 5 bodies: the frame, the upper body, the fork and the two wheels. Only 9 degrees of freedom are considered as the upper body lean is frozen. When the vehicle is simulated with its controller, one differential equation has to be added to the equations of motion to calculate the integral action.

The out-of-plane motion involves the lateral displacement and the yaw and roll angles of the frame, and the handlebar rotation. A satisfying controller has been obtained with the approach presented in the previous section with $Q_u=0.1$, $Q_x=1$ and a weight of 10 on the integral action.

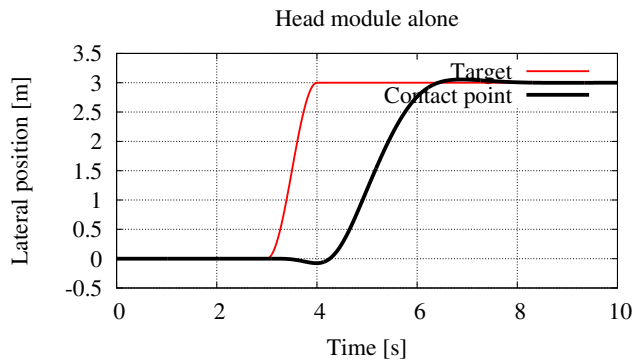


Figure 5. Time history of tracking during the lane change maneuver

Some results obtained with this controller on the head module during a lane change maneuver (deviation of 3 meters distributed over 5 meters) are presented in figures 5 to 8. Due to the shortness of the maneuver, it can be considered as a step response.

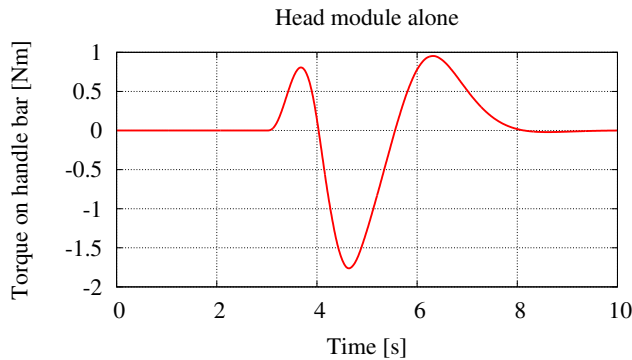


Figure 6. Time history of the torque during the lane change maneuver

Figure 5 shows that the response is reasonably fast and that the steady-state level is reached after a small overshoot. The torque whose time history is presented in figure 6 remains largely in the limits of the human capacities.

From figures 7 and 8, it appears that the yaw and roll angles significantly deviate from zero, which shows that the controller is sufficiently robust to stand the nonlinearities and the coupling with the in-plane motion, which were not taken into account during the design.

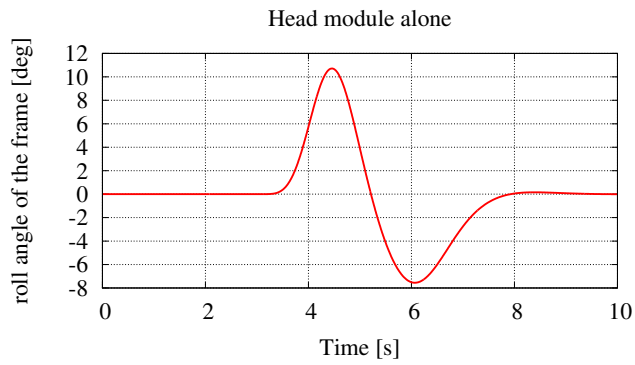


Figure 7. Time history of roll angle during the lane change maneuver

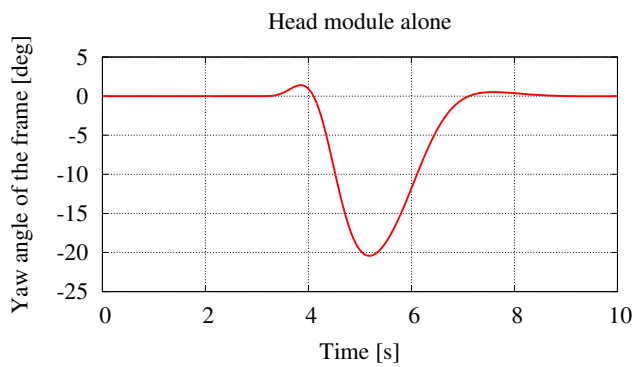


Figure 8. Time history of yaw angle during the lane change maneuver

4.2 The pedal module

In order to be able to manage the pedal module alone, it is attached to a reference body which moves in straight line at a constant speed (figure 9). Physically, this corresponds to the hypothesis that the head module is not affected by the pedal module.

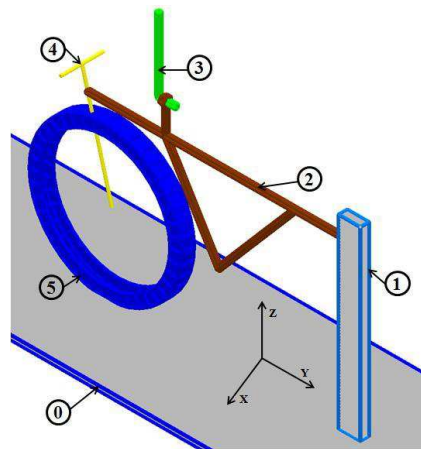


Figure 9. The pedal module alone's model

The multibody model of the pedal module involves 6 degrees of freedom and 5 bodies: the ref-

reference body, the frame, the upper body, the fork and the wheel. Only 5 degrees of freedom are considered as the upper body lean is frozen. One differential equation also has to be added when the controller is involved.

The out-of-plane motion involves the yaw and roll rotation angles at the attachment, and the handlebar rotation. Quite a good controller was obtained on the pedal module alone with $Q_u=0.1$, $Q_x=1$ on the angles, $Q_x=10$ on the angle rates, and a weight of 100 on the integral action. The results of this controller are not presented here as it leads to instability when implemented on the complete *Anaconda*. The retained controller corresponds to $Q_u=0.1$, $Q_x=1$ and a weight of 0.1 on the integral action. With these parameters, the priority is given to the equilibrium, to the detriment of tracking.

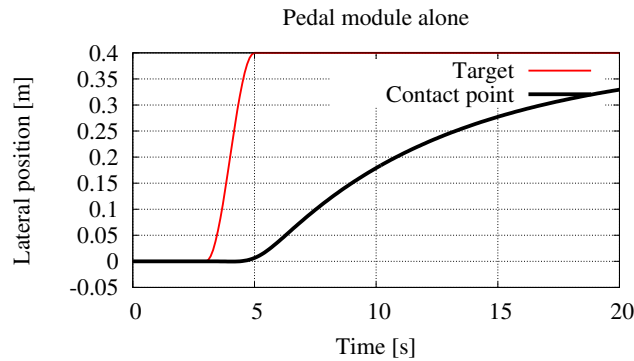


Figure 10. Time history of the tracking of the pedal module alone

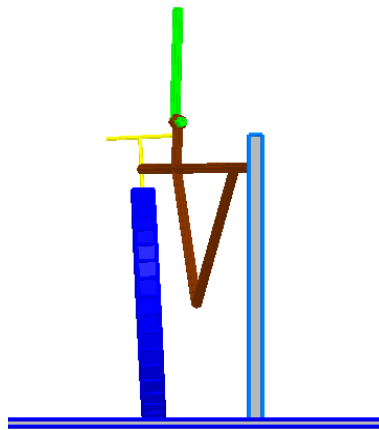


Figure 11. Front View of the pedal module at the end of the lane change

Figures 10 to 14 show the results obtained with the controlled pedal module during a lane change maneuver (deviation of 0.4 meters distributed over 2 meters). Again, it can be considered as a step response.

From figure 10, it appears that the response is very slow: the steady-state value is only asymptotically reached, without any overshoot. The time history presented in figure 12 shows that a steady-state torque is necessary to stabilize the module. This is logical as, to assure the tracking, the pedal module must deviate from the reference body (figure 11). However, it remains humanly

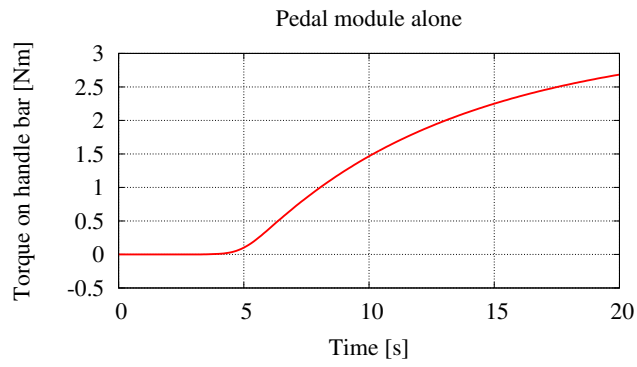


Figure 12. Time history of the torque on the handle bar of the pedal module alone

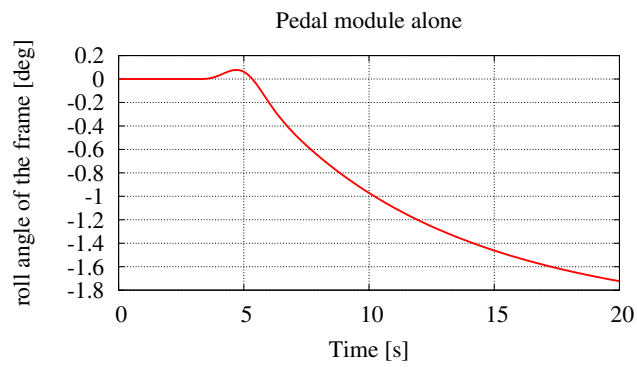


Figure 13. Time history of roll angle of the pedal module alone during maneuver

reachable.

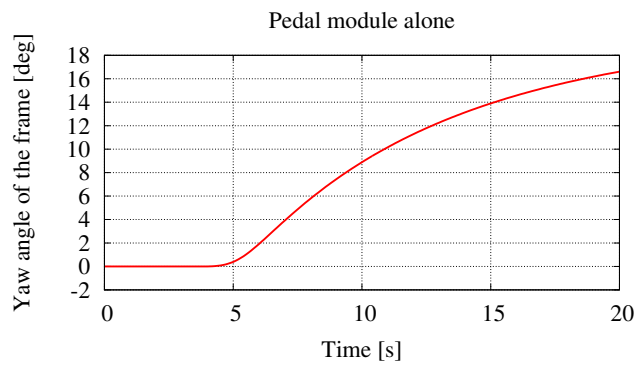


Figure 14. Time history of yaw angle of the pedal module during the maneuver

Again, the yaw and roll angles (figures 13 and 14) take significant amplitudes but the vehicle

remains stable.

This simulation of the pedal module alone is interesting as it shows that it is effectively able to assure by itself its equilibrium and the tracking of a trajectory.

4.3 The *Anaconda* with 2 pedal modules

The controllers designed on the head module and the pedal module alone are now applied as is on an *Anaconda* with 2 pedal modules.

Figures 15 to 18 show the results obtained for the complete vehicle tracking a trajectory corresponding here to a soft lane change maneuver (deviation of 3 meters distributed over 30 meters). The principal observation is that the vehicle is stable, which is confirmed by analysing the poles. The controllers designed on the separated modules remain efficient in the *Anaconda*.

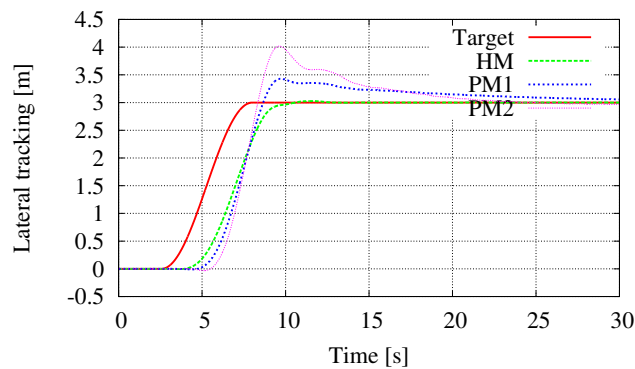


Figure 15. Tracking of the *Anaconda* with 2 pedal modules

The tracking (figure 15) of the head module remains performant and is more or less unaffected by the trailers. On the contrary, as it could be expected from the results on the pedal module alone, the tracking of the pedal modules is only asymptotic but with an overshoot which is larger for the second module.

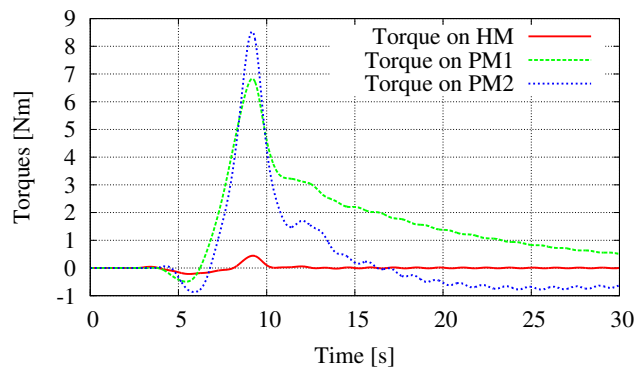


Figure 16. Handlebar torques of the *Anaconda* with 2 pedal modules

The torque on the handlebar (figure 16) of the head module is negligible although the ones on the pedal modules become very important, the one on the last module being a bit larger. Although the values remain humanly reachable, the high values, during a maneuver which can be considered as

soft, show that the pedal modules of the *Anaconda* will require a driver with specific skills.

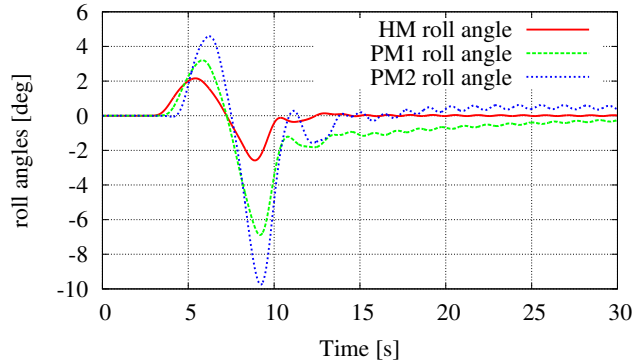


Figure 17. Roll angles in the *Anaconda* with 2 pedal modules

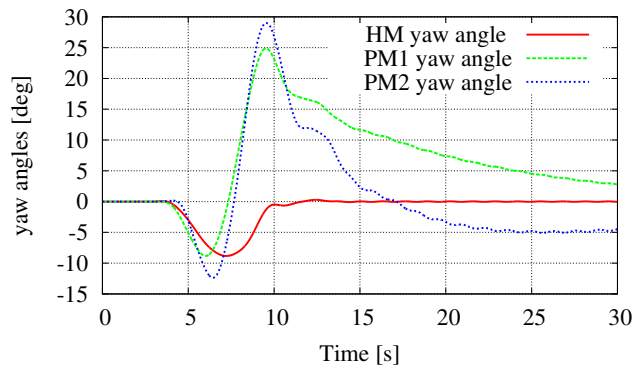


Figure 18. Yaw angles of the *Anaconda* with 2 pedal modules

From figures 17 and 18, it appears that the yaw and roll angles increase from front to rear and are significantly larger for the pedal modules.

4.4 The *Anaconda* with only a regulator on the 2 pedal modules

As the tracking of the pedal modules is anyway very weak, it was decided to build the controller as a regulator trying only to lead the module to the stationary motion in straight line. The tracking is anyway assured by the head module. The regulator was designed on the pedal module alone with $Q_u=0.1$ and $Q_x=1$.

Although it is not specifically included, the tracking is better than with the previous controller (figure 19), with a similar overshoot. This is due to the fact that the regulator makes the pedal modules follow the head one.

The time history of the handlebar torques is smoother but they remain important (figure 20). The persistent oscillations at the end of the simulation also reveal that the system is weakly unstable.

4.5 Rideability index

A system is controllable if all its state variables can be affected by the input. This implies for the *Anaconda* that the steering torques on handlebars influence the time history of all out-of-plane

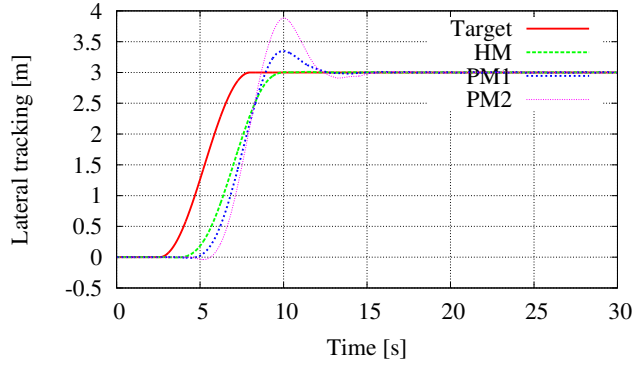


Figure 19. Tracking of the *Anaconda* with a regulator on pedal modules

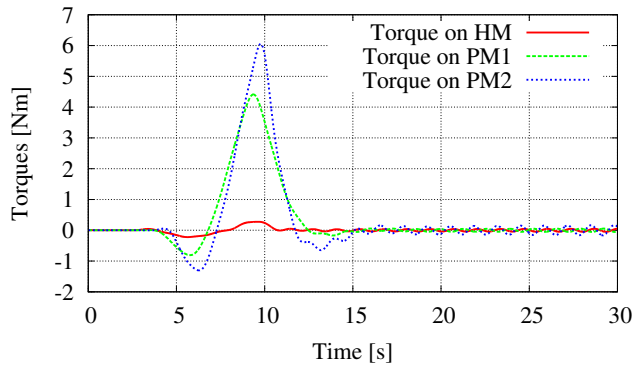


Figure 20. Handlebar torques on the *Anaconda* with a regulator on the pedal modules

state variables. If some states are detached from the steering torques's actions, they will behave outside of any rider's control. From this statement, Seffen[16] proposed a parameter in order to evaluate the riding difficulty for bicycles and motorcycles. The resulting so-called *rideability index* D is defined as:

$$D = \log_{10} \sqrt{\frac{S_{max}}{S_{min}}} \quad (21)$$

where S_{max} and S_{min} are the maximum and minimum singular values of the controllability matrix of the bike/motorcycle. Let us note that the higher the index, the more the two-wheeler is difficult to drive.

As we aim to build an *Anaconda* with a maximum number of modules, we have evaluated, in this first study, the evolution of the rideability index with the amount of modules (figure 21).

We logically observe that the rideability index increases with the amount of pedal modules but the variation becomes less significant from the third pedal module. As far as the rideability index is a reliable indicator of the difficulty of riding, we can hope that people able, after some training, to drive an *Anaconda* with up to two pedal modules should be able to drive a longer specimen.

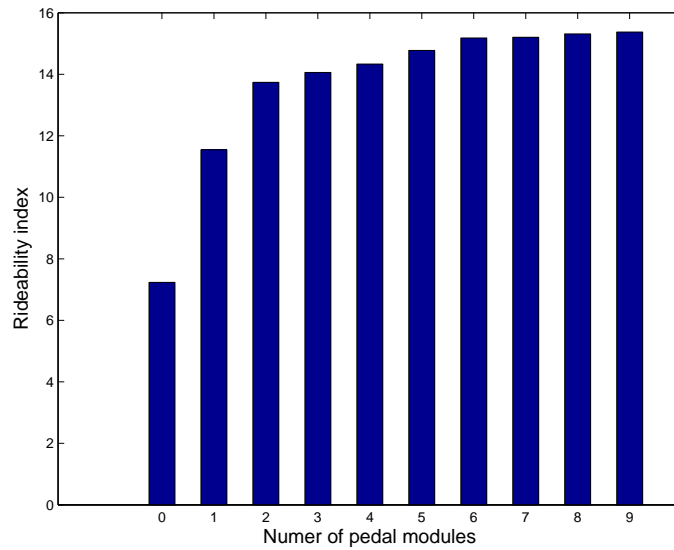


Figure 21. Rideability index evolution with the PM's amount

5 The prototype

In parallel with the study, a prototype was built by students in coachbuilding from the *Lycée Technique Maurice Herlemont* of La Louvière (figure 22). We were very happy to see that, after some training, some young people were able to drive the vehicle and even to follow more or less sophisticated trajectories.



Figure 22. An *Anaconda* prototype

Moreover, some interesting adjustments were made intuitively and contribute to improve the rideability:

- the axis of the fork of the pedal module should be close to vertical;
- the *Anaconda* is easier to drive when the driver sits directly over the fork;
- some masses have been added to lower the center of gravity but the efficiency of this modification has not been rigorously evaluated.

6 CONCLUSION AND PERSPECTIVES

In this paper, a study of the *Anaconda* has been established in order to evaluate its chances to be driven by standard human people. The approach consists in designing controllers separately for the

head module alone and for the pedal module alone and to observe the behaviour of an *Anaconda* with two pedal modules when “equipped” with these controllers. The three mechanical systems have been modelled according to the multibody formalism. The controllers are designed by means of a LQR approach applied on the linearized equations of motion of the multibody model, reduced to the out-of-plane motion. The action of the controller is the torque on the handlebar while the tracked variable is the deviation of the wheel contact point with respect to a given trajectory. With well chosen weights in the LQR process, the *Anaconda* is stable and able to follow a soft lane change although the tracking of the pedal modules is poor. This result is confirmed by an actual prototype which could be driven after some training.

A lot of future prospectives could be cited. Let us mention here a better mechanical and behavioral description of the driver, the analysis of the longitudinal behaviour (braking and propulsion) and the optimization of the geometry for better stability and manoeuvrability.

ACKNOWLEDGEMENTS

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A Listing of the script related to the head module

```
// Title of the application
title := "Simulation of a Bicycle":
nbrdof := 10: // nbrdof : Number of degrees of freedom
nbrbody := 6: // nbrbody : Number of bodies

gravity[1] := 0: gravity[2] := 0: gravity[3] := -9.81:

// Some constants
vini := 5.55556: // initial velocity 20 km/h
epsilon := 18*PI/180: // head angle complement 18 deg
a := 0.76: y1 := 0.345: z1 := 0.765: y2 := 0.950: z2 := 0.700:
y3 := 1.070: z3 := 0.350: y4 := 0.000: z4 := 0.350:
y5 := 0.990: z5 := 0.270: y := 0.300: z := 0.900:

// Inertia characteristics
mass[0] := 1: // ground
Ixx[0] := 1: Iyy[0] := 1: Izz[0] := 1:
mass[1] := 34: // rear frame + rider lower body
Ixx[1] := 4.667: Iyy[1] := 3.869: Izz[1] := 1.272: Iyz[1] := -1.3:
mass[2] := 4: // front frame
Ixx[2] := 0.06: Iyy[2] := 5.843e-2: Izz[2] := 7.5866e-3: Iyz[2] := -9.1192e-3:
mass[3] := 3: // front wheel
Ixx[3] := 0.28: Iyy[3] := 0.1405: Izz[3] := 0.14:
mass[4] := 51: // rider upper body
Ixx[4] := 5.186: Iyy[4] := 4.299: Izz[4] := 1.413: Iyz[4] := -1.444:
mass[5] := 3: // rear wheel
Ixx[5] := 0.28: Iyy[5] := 0.1405: Izz[5] := 0.14:
```

```

// Definition of the position matrices
T0G[0] := Tdisp(0,0,0):
TrefG[1] := Tdisp(q[0],q[1],q[2])*Trotz(q[3])*Trotx(q[4])*Troty(q[5]):
BodyRef[1] := 0:
TrefG[2] := Tdisp(0,0,-0.005)*Tdisp(0,0.5581,0)*Trotx(epsilon)*Trotz(q[6])
           *Tdisp(0,0,-0.07156)*Tdisp(0,0.02610,0):
BodyRef[2] := 1:
TrefG[3] := Tdisp(0,-0.02610,0)*Tdisp(0,0,-0.36994)*Tdisp(0,0.03207,0)
           *Trotx(-epsilon)*Trotx(q[7]):
BodyRef[3] := 2:
TrefG[4] := Tdisp(0,0,-0.005)*Tdisp(0,-0.045,0)*Tdisp(0,0,0.14)*Troty(q[8])
           *Tdisp(0,-0.03,0)*Tdisp(0,0,0.09):
BodyRef[4] := 1:
TrefG[5] := Tdisp(0,0,-0.415)*Tdisp(0,-0.345,0)*Trotx(q[9]):
BodyRef[5] := 1:

// Initial conditions
qi[2] := 0.72:
qdi[1] := vini: qdi[7] := -vini/z3: qdi[9] := -vini/z4:

// Simulation parameters
FinalTime:=30: StepSave:=0.01: StepMax:=0.001:

// Set simplify to 1 if you want the expressions to be simplified by the
// symbolic engine
SIMPLIFY:=1:
// Set FORCES to 1 in case you want to include *.AppEff.cpp into procedure
// AddAppliedEfforts() to define forces other than gravity
FORCES:=1:
// Set ANIM to 1 in case you want CaGEM to generate the skeleton code
// for visualization and animation of the system
ANIM:=1:
// Set STATIC to 1 in case you want CaGEM to generate the code
// to search for static equilibrium before integration
STATIC:=1:
// Set PLOT to 1 in case you want CaGEM to generate the GNUPLOT code
// to plot the evolution of position, velocity and acceleration
PLOT:=1:
// SET LATEX_FR to 1 if you want the LaTeX report in French
LATEX_FR:=0:
// SET LATEX_EN to 1 if you want the LaTeX report in English
LATEX_EN:=0:

```