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Assessing slip of a rolling disc and the implementation of a tyre model in the benchmark bicycle

E.J.H. de Vries[*], J.F.A. den Brok[*#]

[*] Laboratory for Engineering Mechanics
Delft University of Technology
Mekelweg 2, NL-2628 CD Delft, The Netherlands
e-mail: E.J.H.devries@tudelft.nl

[#] currently:
ASML Netherlands B.V.
De Run 6501, NL-5504 DR, Veldhoven
j.f.a.denbrok@gmail.com

ABSTRACT

For a motorcycle simulator the development of a real time bicycle/motorcycle model with a compact formulation of the dynamic behaviour is required. The modeling environment MatLab SimMechanics was posed as a prerequisite. No suitable tyre model is available in the library of this software, the tyre model has been developed using linear tyre slip characteristics, and point contact of knife-edged wheels with the ground. The simulation in simmechanics of a single wheel with this tyre model stresses the importance of the propagation speed of the tyre contact point. Two wheels have been assembled with two frame bodies to build the Wipple bicycle in SimMechanics. The simulation model has been validated against the bicycle benchmark using eigenvalue analysis. The eigenvalues of SimMechanics model were obtained from curve fitting a (complex-) exponential function to a time response from simulation. The match is remarkable despite of the tyre models included in the simulation model.

Keywords: Euler disc, bicycle wheel, tyre model, SimMechanics model.

1 INTRODUCTION

Recently the use of simulators has become increasingly popular. The vehicle motion is replicated by a motion (stewart-) platform, and the scenery is visualized on screens connected to the cockpit. The application area of these simulators is quickly expanding from airplane pilot training to race driver training and even game-arcades. The industrial partner in this research, Cruden considers the development of a motorcycle simulator. For a motorcycle simulator the development of a real time bicycle/motorcycle model with a compact formulation of the dynamic behaviour is required. The modeling environment MatLab SimMechanics was posed as a prerequisite.

2 WHEEL MODEL

From the SimMechanics library Body element has been used as the basis for the wheel module. Though some examples of rolling spheres are present in the examples library, no real wheels with non-holonomic constraints were made available. In the future application of a real time bicycle or motorcycle model eventually the tyre slip angle will play a role as force generating input to the tyre model, so we immediately start modeling non-perfect rolling.
For the development of the tyre model the wheel was considered as a disc with knife edge contact to the ground. A tyre model typically uses slip quantities as the input and calculates forces and moments as outputs. Longitudinal- and lateral slip are calculated as the components of a normalized slip velocity in the contact point. The location of this contact point can be denoted with a position vector **r** pointing from the wheel disc centre to the contact point.

The assessment of the slip quantities has been based entirely on the vector calculation presented by Pacejka in [1] the contact point can be found in radial direction $\mathbf{e}_r$ at a scalar distance r from the wheel centre. Here $\mathbf{e}_r$ is recursively defined as being orthogonal to the axial $\mathbf{e}_s$ and longitudinal direction $\mathbf{e}_l$, where the longitudinal direction is the intersection of the road plane and wheel plane, thus the vector orthogonal to road normal $\mathbf{n}$ and axle direction $\mathbf{e}_s$. See Figure 1

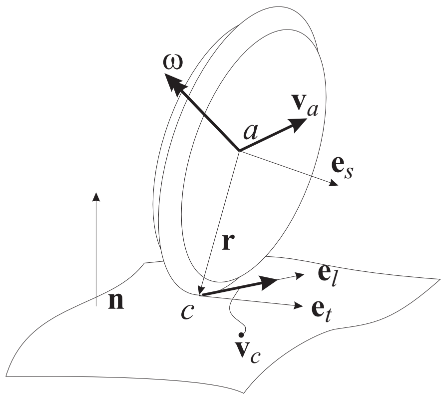

Figure 1. The wheel disc with contact point location r

The contact point relative position vector results from the following equations. By taking the cross product of the rotated wheel axle $\mathbf{e}_s$ and road normal $\mathbf{n}$ one gets the longitudinal vector, $\mathbf{l}$:

$$\mathbf{l} = \mathbf{n} \times \mathbf{e}_s \tag{1}$$

$\mathbf{e}_l$ denotes the orthonormal (orthogonal and unit length) vector in longitudinal direction.

$$\mathbf{e}_l = \frac{\mathbf{l}}{\|\mathbf{l}\|} \tag{2}$$

Using the cross product of the road normal and longitudinal vector $\mathbf{e}_l$ results in the lateral direction $\mathbf{e}_t$. The angle between longitudinal and normal is $90°$ and thus the result of their cross product is automatically unit length.

$$\mathbf{e}_t = \mathbf{n} \times \mathbf{e}_l$$

In order to calculate the wheel radial direction $\mathbf{e}_r$, the cross product of the longitudinal and current wheel axle vector is used. $\mathbf{e}_r = \mathbf{e}_l \times \mathbf{e}_s$ The wheel radial direction is multiplied with the scalar length r to provide the radius vector $\mathbf{r}$:

$$\mathbf{r} = r\left(\mathbf{e}_l \times \mathbf{e}_s\right) \tag{3}$$

Where $\mathbf{r}$ denotes the position vector of the contac point c from wheel centre, and r is the scalar distance from the wheel centre to the point of contact. The global position of contactpoint c can now be found from the position of the wheel center $\mathbf{x}$, and $\mathbf{r}$. the difference between the road coordinate and contact point position can be projected on the normal direction. This is a measure of the tyre deformation, and allows calculation of the normal load. Assuming for convenience that the road surface is a plane through (0,0,0) we find:

$$d = \left(\mathbf{x}_a + \mathbf{r}\right) \cdot \mathbf{n} \tag{4}$$

With the motion of the centre of the wheel disc and the vector $\mathbf{r}$ available, the derivation of the velocity of the material point s on the wheel, momentarily in the contact point c, is straightforward:

$$\mathbf{v}_s = \mathbf{v}_a + \boldsymbol{\omega} \times \mathbf{r} \tag{5}$$

A linear spring-damper can now be used to calculate the force in vertical (normal to the road) direction. The velocity of the point s needs to be normalized with the wheel speed v_x, to obtain the slip quantities α and κ that are the inputs for the tyre horizontal force model.

$$\lambda = \frac{\mathbf{v}_s}{v_x}, \quad \tan\alpha = -\lambda_y, \quad \kappa = -\lambda_x \tag{6}$$

A simple linear tyre model defines the dissipative tyre contact forces, proportional to the established slip quantities.

$$F_x = C_{F_\kappa}\kappa\,, F_y = C_{F_\alpha}\alpha$$

However normalizing slip with the wheel-centre longitudinal velocity $v_x \rightarrow v_{a,x}$ is practical, but incorrect. We show that the singularity caused by dividing by zero axle speed, poses difficulties for the numerical solvers when simulating the wheel as an Euler disc. The behaviour of an Euler disc, in practice comparable to a spinning coin on a tabletop, has many instances where the centre of gravity has momentarily zero speed while the disc is still rolling. Dividing by near-zero speed causes the dynamics to stiffen progressively, causing poor numerical performance of the solver.
The correct velocity to use in the normalization of slip is the propagation speed of the contact point $v_{c,x}$, which comprises $v_{a,x}$ and a term with $\dot{\mathbf{r}}$. Since $\mathbf{r}$ is the result of sequential cross products, its time derivation is awkward, yet possible.

2.1 The propagation speed v_{cx}

Figure 2 illustrates the difference between the longitudinal velocity of the contact point and the velocity of the centre of gravity. Clearly this difference is small for wheels small camber or yaw rate. The combination of both variables is exaggerated in the example of the Euler disc motion, that serves as the test scenario for the development of a single wheel model.

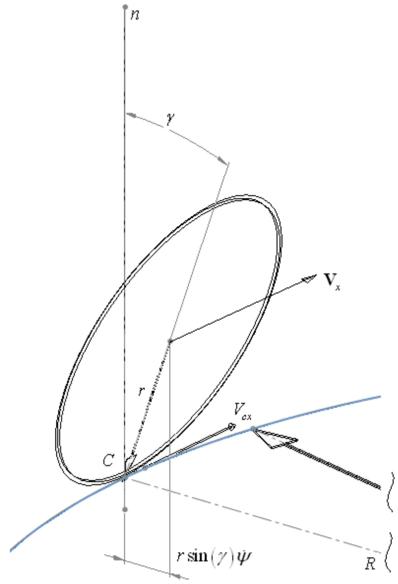

Figure 2 The wheel disc under camber angle γ, illustration of the difference between the path of c and of the wheel centre a

When the global position of c is given by $\mathbf{x}_c = \mathbf{x}_a + \mathbf{r}$ and the velocity of the wheel body $\dot{\mathbf{x}}_a = \mathbf{v}_a$ can be obtained from the SimMechanics environment, we now concentrate on the derivation of $\dot{\mathbf{r}}$:

$$\dot{\mathbf{r}} = r\left(\dot{\mathbf{e}}_l \times \mathbf{e}_s + \mathbf{e}_l \times \dot{\mathbf{e}}_s\right) \tag{7}$$

Equation (7) show that for the radial direction time derivative $\dot{\mathbf{e}}_r$ the derivative of the longitudinal direction vector $\dot{\mathbf{e}}_l$ is needed. This vector is the result of a cross product itself and it is important to notice that this vector has been normalized to give the vector unit length. Knowing that $\mathbf{e}_s$ and $\mathbf{n}$ are both unit length, we can write $\|\mathbf{l}\| = \sin\theta$ where θ is the smallest angle between the vectors of the road-normal vector and wheel-axle. The normalization is time dependent since the camber angle varies. Note that $\theta + \gamma = 90°$.

$$\mathbf{e}_l = \frac{\mathbf{l}}{\|\mathbf{l}\|} = \frac{\mathbf{l}}{\sin\theta} \tag{8}$$

With (8) we find for $\dot{\mathbf{e}}_l$:

$$\dot{\mathbf{e}}_l = \frac{\dot{\mathbf{l}}\sin\theta - \mathbf{l}\cos\theta\dot{\theta}}{\sin^2\theta} = \frac{\dot{\mathbf{l}}}{\sin\theta} - \mathbf{e}_l\cot\theta\dot{\theta} \tag{9}$$

In expression (7) $\dot{\mathbf{e}}_s$ is required; the time derivative of the rotated wheel axle, its calculation evolves similar to the time derivative of a position vector connected to the wheel body.

$$\dot{\mathbf{e}}_s = \boldsymbol{\omega}\times\mathbf{e}_s \tag{10}$$

This result leads to the following $\dot{\mathbf{l}}$:

$$\dot{\mathbf{l}} = \dot{\mathbf{n}}\times\mathbf{e}_s + \mathbf{n}\times(\boldsymbol{\omega}\times\mathbf{e}_s) \tag{11}$$

For the second term in equation (11) we can use the vector 'triple product':

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = \mathbf{b}(\mathbf{a}\cdot\mathbf{c}) - \mathbf{c}(\mathbf{a}\cdot\mathbf{b}) \tag{12}$$

That transforms (11) into:

$$\dot{\mathbf{l}} = \dot{\mathbf{n}}\times\mathbf{e}_s + \boldsymbol{\omega}(\mathbf{n}\cdot\mathbf{e}_s) - \mathbf{e}_s(\mathbf{n}\cdot\boldsymbol{\omega}) \tag{13}$$

And finally $\dot{\mathbf{l}}$ substituted in (9) leads to:

$$\dot{\mathbf{e}}_l = \frac{\left(\dot{\mathbf{n}}\times\mathbf{e}_s + \boldsymbol{\omega}(\mathbf{n}\cdot\mathbf{e}_s) - \mathbf{e}_s(\mathbf{n}\cdot\boldsymbol{\omega})\right)}{\sin\theta} - \mathbf{e}_l\cot\theta\dot{\theta} \tag{14}$$

Now $\dot{\mathbf{e}}_0$ found from (10) can be substituted in

$$\dot{\mathbf{e}}_r = \dot{\mathbf{e}}_l\times\mathbf{e}_s + \mathbf{e}_l\times(\boldsymbol{\omega}\times\mathbf{e}_s) \tag{15}$$

Substitution of (14) in (15) an using the general relation (12) for the triple product results in

$$\dot{\mathbf{r}} = r\left\{\frac{\left(\dot{\mathbf{n}}\times\mathbf{e}_s + \boldsymbol{\omega}(\mathbf{n}\cdot\mathbf{e}_s) - \mathbf{e}_s(\mathbf{n}\cdot\boldsymbol{\omega})\right)\sin\theta - \mathbf{l}\cos\theta\dot{\theta}}{\sin^2\theta}\times\mathbf{e}_s + \boldsymbol{\omega}\left(\frac{\mathbf{l}}{\sin\theta}\cdot\mathbf{e}_s\right) - \mathbf{e}_s\left(\frac{\mathbf{l}}{\sin\theta}\cdot\boldsymbol{\omega}\right)\right\}$$

Since we are interested in the propagation speed v_{cx}, which has its contribution in the longitudinal direction, the derivative of the wheel radius vector has to be projected onto the longitudinal vector.

$$\dot{\mathbf{r}}\cdot\mathbf{e}_l = r\left[\left\{\frac{\left(\overbrace{\dot{\mathbf{n}}\times\mathbf{e}_s}^{1} + \boldsymbol{\omega}(\mathbf{n}\cdot\mathbf{e}_s) - \overbrace{\mathbf{e}_s(\mathbf{n}\cdot\boldsymbol{\omega})}^{2}\right)}{\sin\theta}\times\mathbf{e}_s - \cos\theta\dot{\theta}\overbrace{\mathbf{e}_l\times\mathbf{e}_s}^{5} + \boldsymbol{\omega}\left(\overbrace{\frac{\mathbf{l}}{\sin\theta}\cdot\mathbf{e}_s}^{3}\right) - \overbrace{\mathbf{e}_s\left(\frac{\mathbf{l}}{\sin\theta}\cdot\boldsymbol{\omega}\right)}^{4}\right\}\cdot\mathbf{e}_l\right] \tag{16}$$

In order to simplify the above equation we can use some vector relations and general properties.

1. The derivative of n on flat level roads is zero: erasing $\dot{\mathbf{n}}\times\mathbf{e}_s$
2. $\mathbf{e}_s\times\mathbf{e}_s \triangleq 0$ therefore we lose term ^{2} in (16)

3. 4. The dot product of two orthogonal vectors is always zero: $\mathbf{e}_l \cdot \mathbf{e}_s \triangleq 0$, which allows to erase [3] and [4].
5. $\cot\theta\dot{\theta}\left(\mathbf{e}_l \times \mathbf{e}_s\right)\cdot \mathbf{e}_l = \cot\theta\dot{\theta}\mathbf{e}_r \cdot \mathbf{e}_l = 0$

Finally we can assemble the correct propagation speed to calculate the slip ratios

$$\dot{\mathbf{r}} \cdot \mathbf{e}_l = r\left[\left(\frac{\boldsymbol{\omega}\left(\mathbf{n}\cdot\mathbf{e}_s\right)}{\sin\theta}\times\mathbf{e}_s\right)\cdot\mathbf{e}_l\right] = \left[\left(\frac{\boldsymbol{\omega}\left(\cos\theta\right)}{\sin\theta}\times\mathbf{e}_s\right)\cdot\mathbf{e}_l\right] = \left[\left(\frac{\boldsymbol{\omega}\left(\sin\gamma\right)}{\|\mathbf{l}\|}\times\mathbf{e}_s\right)\cdot\mathbf{e}_l\right] \tag{17}$$

All variables in (17) are eiter well defined, or can be extracted from de body module in sim mechanics by means of a sensor block.
The lateral slip or slipangle α is defined as the ratio of lateral slip speed v_{sx} and the forward speed v_{cx} of the tyre contact point. Similarly for longitudinal slip κ:

$$\tan\alpha = \frac{-v_{sy}}{\left|v_{cx}\right| + \varepsilon}, \quad \kappa = -\frac{-v_{sx}}{\left|v_{cx}\right| + \varepsilon} \tag{18}$$

The calculations presented will be included in the simulation environment by plane simulink modelling. Figure 3 shows three successive snapshots of the simulated single wheel motion, when it has a low speed and behaves as the Euler disc. In Figure 4 the longitudinal velocity of the centre of gravity is presented in the left axis and the propagation velocity of the contact point is presented in the right axis. The single wheel has been further elaborated by including relaxation behaviour for both longitudinal and lateral slip. The relaxation behaviour substitutes equation (18) as the slip definition, and allow simulation at zero speed, without the need for introducing ε to prevent singularity. For the relaxation equations we refer to [1] and [3]. The full wheel model in Simulink is given by a snapshot in the appendix

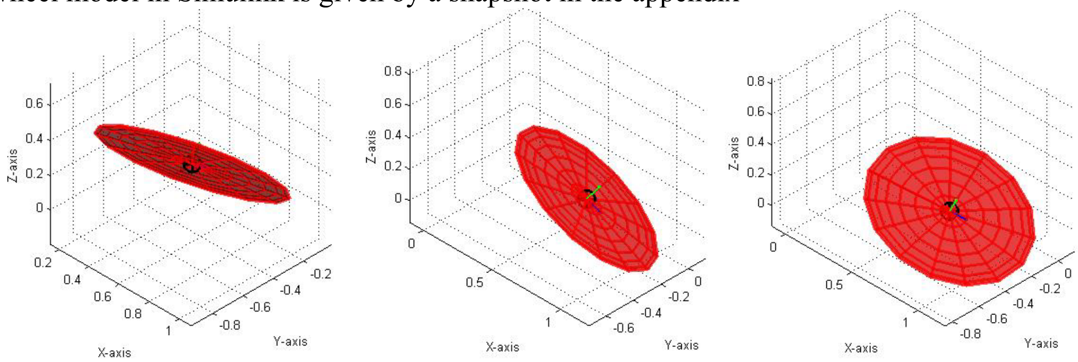

Figure 3 Three successive snapshots of the SimMechanics visualisation of the single wheel simulated as Euler disc.

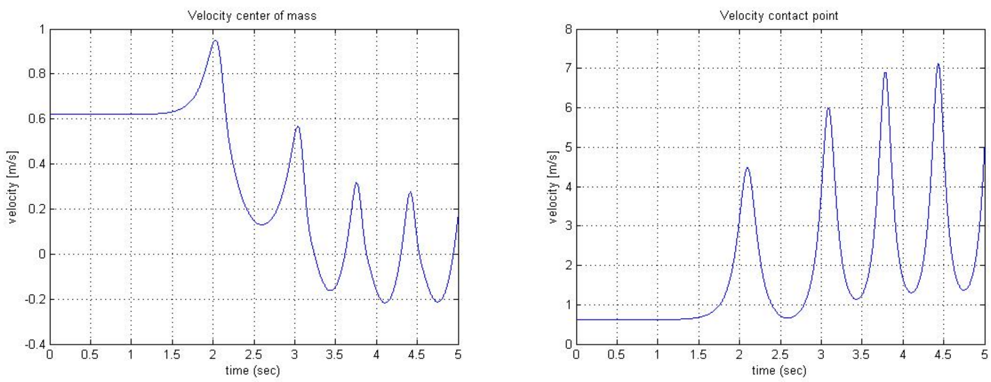

Figure 4 Velocity as a function of time. Simulated with a initial forward velocity of 0.62 [m/s] centre of mass *a* and contact point *c* respectively.

It is important to mention that the angle-coordinates that SimMechanics uses depend on the type of constraint that is selected to connect to the body. The generic ‘custom joint’ allows all degrees of freedom to be specified by the user in the GUI presented by Figure 5. The default choice suggested is assigning Euler angles in *xyz* sequence. In vehicle dynamics the vehicle typically drives in *x* direction thus the wheels experience large angles about the *y* axis. The *xyz* sequence of Euler angles should be avoided at all cost, the Euler angles experience so called gimball-lock after a wheel rotantion of 90°. After discovering this we use *zxy* as the order of Euler angles, or even better use the ‘six-DOF’ constraint that assigns quarternion coordinates to the wheel-body

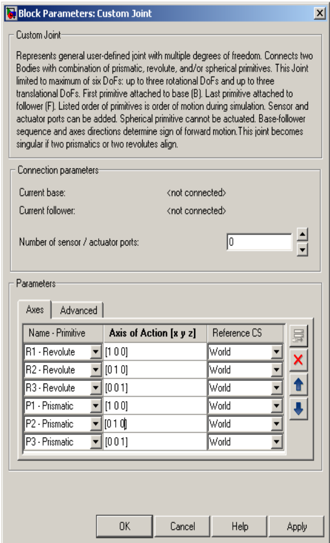

Figure 5 The SimMechanics graphical interface (GUI) that allows custom degrees of freedom to be assigned to the body

3 BICYCLE MODEL

In this section the benchmark bicycle is introduced though it has been taken from the literature [2] in subsection 3.1, then the SimMechanics model is explained in subsection 3.2. The results are resented by some time response plot. Time response generated at various velocities and with saddle and steer excitation have been post-processed to create eigenvalue versus speed diagram. Both types of results are presented in subsection 3.3

3.1 The bicycle benchmark

The so called Whiple bicycle is the most basic mechanical model of a bicycle is described in [2]. The model consists of four rigid bodies, i.e. the rear frame with the rider body rigidly attached to it, the front frame being the front fork, the front an rear wheels. Due to its validate equations and standardised parameterset from [2] it serves as the benchmark to verify and validate the SimMechanics model.

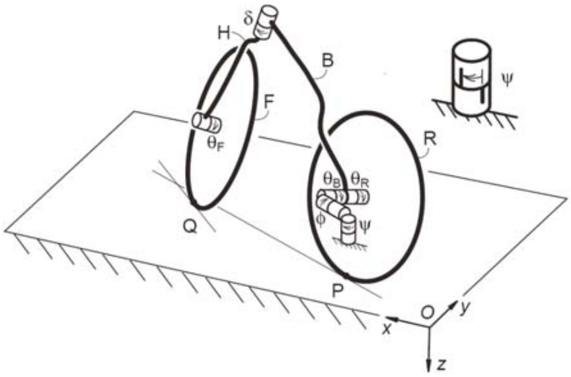

Figure 6: The 'Whipple' or benchmark bicycle, picture taken from [2]

The bicycle benchmark is fully characterized by 25 parameters. Most numerical values are representative for real bicycles. The eigenvalues of the uncontrolled bicycle and their corresponding eigenvectors allows the analysis of the bicycle dynamics.

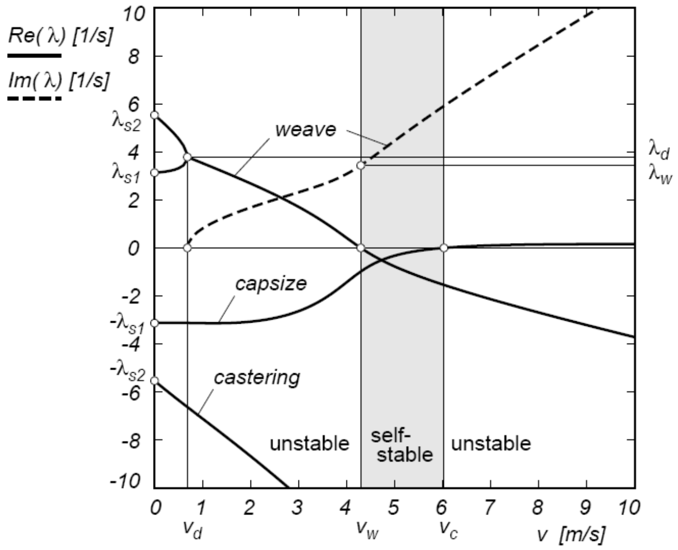

Figure 7: Eigenvalues λ from the linearized stability analysis for the benchmark bicycle The solid lines correspond to the real part of the eigenvalues and the dashed line corresponds to the imaginary part of the eigenvalues, in the forward speed range of $0 \leq v \leq 10$ m/s . graph taken from [2]

At low speeds, starting at zero, the eigenvalues come in two positive and negative pairs and represent the instability of an inverted pendulum. At sufficiently higher speed, the two positive real eigenvalues commonly merge to form a complex conjugate pair with positive real parts. This represents unstable oscillatory motion and is referred to as the weave mode. The bicycle leans and steers from side to side. As forward speed increases, the frequency of this weave increases, as is indicated by the increasing magnitude of the imaginary parts of the complex conjugate eigenvalues. This increase in magnitude becomes nearly linear with the increase in forward speed, and so the wavelength of the weave is nearly constant.

At even higher speeds this pair crosses the real axis and the weave motion becomes stable. This is the beginning of the range of forward speeds for which the bicycle is self-stable. The smallest of the two initially-negative eigenvalues corresponds to the capsize mode. it becomes positive (unstable) at a speed above the weave speed, marking the end of the self-stable range of speeds. Finally, the eigenvalue initially most negative has an eigenvector dominated by steer rate and represents the castor mode: the tendency of the front wheel to steer in the direction the bicycle is moving. It becomes more stable as forward speed increases.

3.2 SimMechanics model

In SimMechanics the bicycle has been modelled by assembling a main frame body, wich includes the rigid rider to a front fork body by a revolute joint. Both frame parts contain a wheel that again is connected to the frame by revolute joints that represent the wheel axles. The newly developed tyre model interfaces with the wheel body using a sensor and actuator connection. A coarse impression of the model is given in Figure 8
After the bicycle model was build a few simulations were performed. However regardless the initial speed or perturbing force the bicycle became instantly unstable. At first we thought the error could be found in a misinterpretation of a sign convention of the front and rear wheel. e.g. introducing a plus in the rear wheel and a correct minus sign in the front wheel configuration, could result in a self exciter of the rear end. But this was not the case. Since the error could not be found in the bicycle configuration or parameters, it was presumably caused in the tyre model.

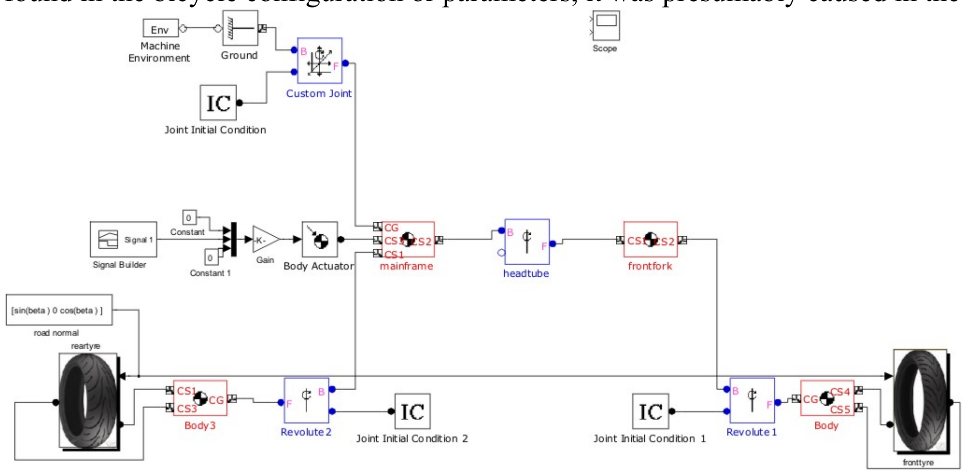

Figure 8 Overview of the SimMechanics model structure with its four bodies as red blocks, the interconnecting revolute joints presented as blue blocks, and a dedicated front and rear wheel module.

For single wheel simulations initially a rotational damper (around the z-axis) was build in the wheel to increase the energy dissipation in the tyre. The single wheel shows the tendency to perpetual behaviour. Removal of this damping was the key to a successful simulation.
Another possibility is the use of a damper in combination with a spring, in other words ad another relaxation system, Just as it has been used for the longitudinal and lateral force. Refer to [3],[4] for the relaxation model for turnslip.

3.3 Bicycle simulations

The variables that we wanted to measure for an adequate validation are δ the steer angle, $\dot{\delta}$ the steer angle rate, γ the lean (roll) angle, $\dot{\gamma}$ the lean (roll) angle rate, and v the forward speed (which would range from 0 to 10 m/s). With these variables we could then compare the measured values to the calculated values. Below four characteristic speeds are discussed in more detail. In each case the lean and steer rate is shown, since the lean rate was a very compact figure and needed some up scaling.

First the unstable weave speed is taken followed by the stable weave.

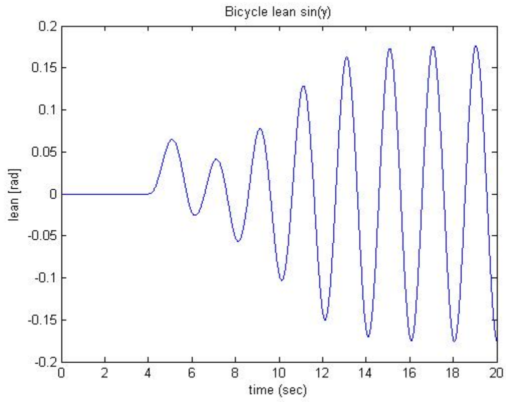

Figure 9 Bicycle lean versus time at an unstable weave velocity v=4.0 [m/s].

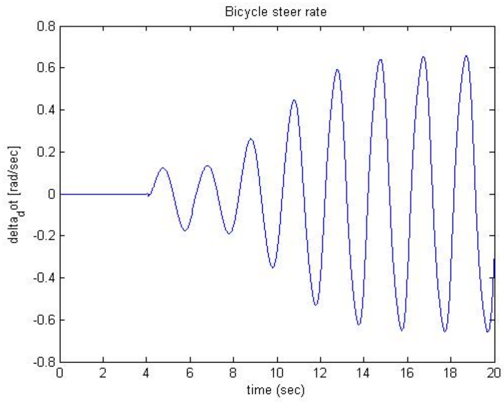

Figure 10 Bicycle steer rate versus time at an unstable weave velocity v=4.0 [m/s]

Figure 9 and versus timeFigure 10 represents the unstable oscillatory motion and is referred to as the weave mode. The bicycle leans and steers from side to side. The increasing (undamped behaviour) for lean and steer are in accordance with the linearized benchmark model since this speed is located in the unstable speed region.

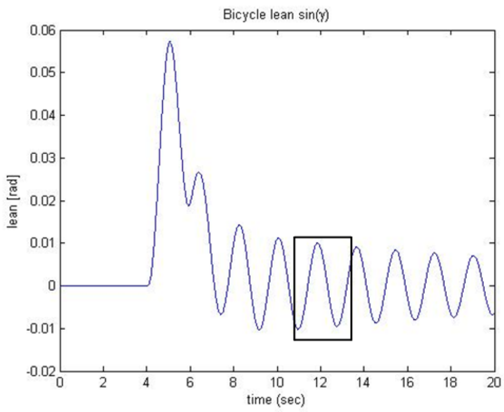

Figure 11 Bicycle lean versus time at a stable weave velocity v=4.5 [m/s].

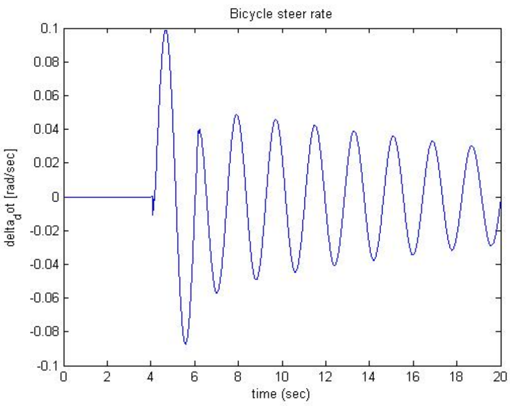

Figure 12 Bicycle steer rate versus time at a stable weave velocity v=4.5 [m/s].

Figure 11 shows a simulation of the weave speed at approximately 4.5 m/s. After the perturbing force the bicycle shows a slightly damped oscillatory behaviour. The same holds for the steer rate. As this typically speed is located in the stable speed region of the bicycle this oscillatory behaviour will damp out.

Since we had limited confidence in the automated linearization within SimMechanics especially in including the negative stiffness generated by gravity, we perform time simulations, the behaviour of the bicycle is disturbed by a pulse like force or torque. The response of the lean angle and steer angle rate have been postprocessed to extract the eigen values: The standard (complex)exponential function was fitted with a least squares optimization to the simulation data, after a suitable segment of measurement data has been selected. An example is the black box of Figure 6. More details on the identification procedure can be found in [5]

$$y = C_0 + C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \tag{19}$$

In **Figure 13** the identified eigenvalues have been plotted against the velocity, in the same fashion as the literature reference [2] shows. The open circles are the result of identification of the complex exponential function on the simulated time response obtained from the simmechanics model. The bicycle behaviour that has been excited by a saddle force pulse is represented with black circles. The higly damped weave mode has been excited with a steer torque pulse, the

identified imaginary part of the eigenvalue is plotted with red circles. The linear benchmark model results in eigenvalues that are represented with solid dots.
The result is very convincing, most circles coincide with the dots. Despite the presence of a tyre model with the associated slip and dissipation the uncontrolled bicycle dynamics as still very similar to the benchmark bicycle with the perfect rolling (no-slip) wheels. The velocity was increased with 1 ms^{-1} steps. The speed where stable behaviour is first found was found by manual iteration with the SimMechanics model. The bounds found for the stable speed range: 4.3 ms^{-1} and 6.0 ms^{-1} match the benchmark model up to three digits. The difference can be increased when other bicycle tyre parameters are included. Preferably the new values should be supported by measurements on bicycle tyres.

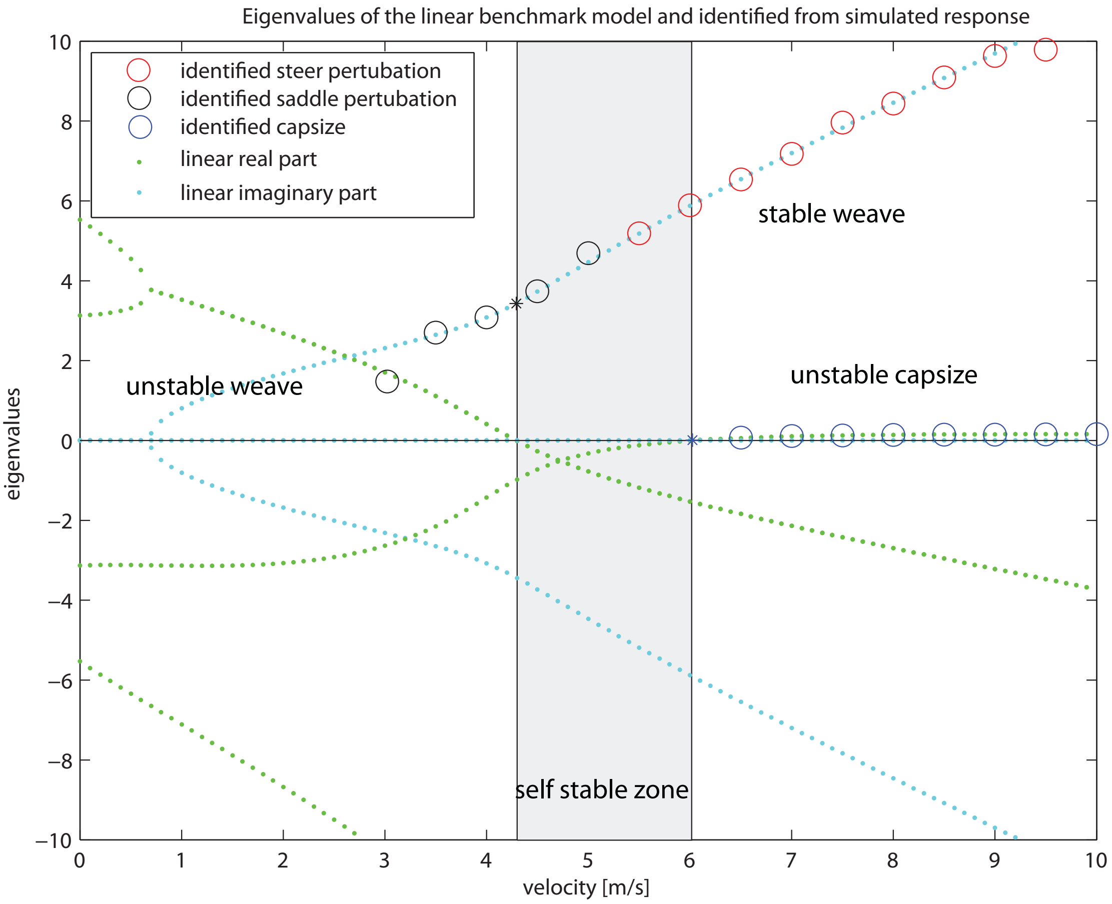

Figure 13 The eigenvalue versus speed for both benchmark bicycle and the SimMechanics

4 CONCLUSIONS

A linear tyre model has been developed in SimMechanics environment, The tyre model directly connects to the Wheel body. A single wheel can be successfully simulated, up to extreme camber angles resembling the behaviour of an Euler disc. There is no need for a wheel carrier body. Simulating a single wheel stresses the importance of the determination of the propagation speed of the tyre contact point, which is awkward, yet possible. The definition of the angle coordinates of a single body with six degrees of freedom requires caution in assigning the Euler angle sequence. SimMechanics presents *xyz* sequence as default, while *zxy* should be used.
By connecting two wheels to a front-fork and a rear-frame bodies with revolute joints one easily builds a Wipple bicycle. The behaviour of the SimMechanics model was validated against the bicycle benchmark. Despite the presence of a tyre model with the associated slip and dissipation the uncontrolled bicycle dynamics as still very similar to the benchmark bicycle with the perfect rolling (no-slip) wheels. Introducing of spin-damping/friction leads to instability unless compliance is added with a relaxation equation.

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