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On Linear-Parameter-Varying Roll Angle Controller Design for Two-Wheeled Vehicles

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ABSTRACT

The work addresses the design of a roll angle controller for a motorcycle simulator. The lean angle controller is part of a higher level virtual rider that plans and executes the trajectory. The higher level controller generates a reference roll angle that the inner roll angle controller tracks. The proposed controller is a Linear Parameter Varying (LPV) controller. Jacobian linearization of a multi-body vehicle simulator is employed to obtain an LPV model that is then used to design the controller; in particular it is shown that the dynamics are strongly dependent on the longitudinal velocity and lateral acceleration. Simulation tests and comparison with a fixed structure controller shows that the LPV controller by adapting to the varying dynamics achieves better performance.

Keywords: motorcycle dynamics, lean angle controller, Linear-parameter-varying systems

1 INTRODUCTION

The acceptance of vehicle dynamics control systems in four-wheeled vehicles has been increasing for at least a decade now. Nowadays systems like Anti Locking Braking (ABS), Traction Control (TC) and Electronic Stability Control (ESP) not only render the car safer but also help the driver increasing his/her performance on the racing track.

Two-wheeled vehicles manufacturers were at first reluctant to adopt these technologies. This happened mainly for economic, “romantic” and technological reasons. Motorcycle manufacturers have less resources to invest in R&D. Further, typically high-end motorcycle riders consider their motorcycles recreational vehicles and “do not want any help riding their bikes”. While this is mostly a cultural obstacle, its roots are deep and touch technological issues. The dynamics of two-wheeled vehicles are more complex than that of four-wheeled vehicles and the technologies developed for four-wheeled vehicle are not directly transferable to motorcycles ([8]) and so the first attempts at developing control systems were not always successful. In the past few years this tendency has changed; high-end motorcycles are now being equipped with performance-oriented electronic control systems (like racing ABS [2], [20], traction control [9], semi-active suspensions [10, 22], semi-active steering dampers [11] and so on). The change of attitude is mainly due to the success of these systems on the racing track. The development has gone as far as presenting some preliminary results on electronic stability control of two-wheeled vehicles [15, 21].

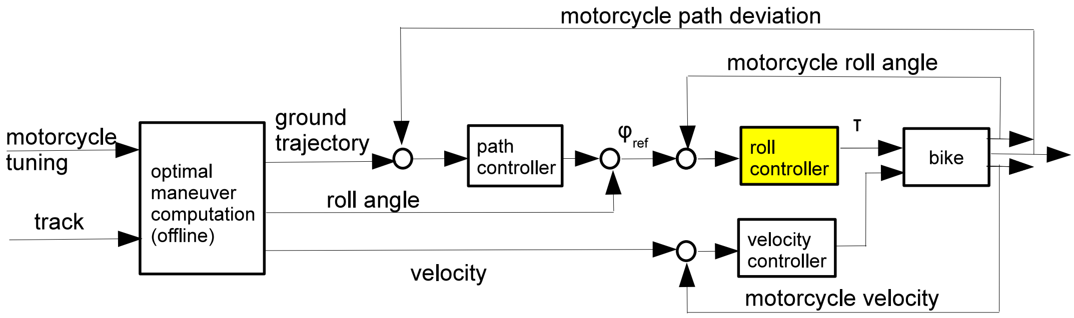

Figure 1. Optimal Maneuver Method block diagram. Notice the decoupled velocity and acceleration control loops.

One of the problems that developers of electronic stability control systems for two wheeled-vehicle face is the difficulty to safely test their prototypes. A car can be easily rigged to safely test ESC systems in extreme conditions; the same cannot be achieved for a motorcycle as the rider is always exposed. Vehicle dynamic simulation is a solution to this problem.

Accurate vehicle simulators can help the initial development and testing of these systems; they are also helpful in the mechanical design of motorcycles and in optimizing the vehicle tuning for a specific track in view of a race, for example. In the past several years, strong efforts have been put forward to derive accurate mathematical models of two-wheeled vehicles [1, 14, 12]. Accurate simulation of two-wheeled vehicle dynamics is only one aspect of the problem; the other aspect is the rider. Two solutions have been proposed: on one hand it is possible to design simulators with human machine interfaces so that the human is actually driving the car (see for example [13]), the drawback of this approach is that in order to guarantee real-timeness the dynamic model has to be simplified; the second approach consists in developing models also for the driver. In this way it is possible to use the complete dynamical model and to achieve repeatable results.

The literature on two-wheeled vehicle rider models is recent [3, 6, 7]. In most cases a two-layer controller is adopted: an *external* control law computes the control input that would track the reference ground trajectory, and an *inner controller* stabilizes the dynamics. In [4, 5], an extra step is added where an optimal trajectory is computed. The *Optimal Maneuver Method* is used to compute the reference ground path and speed to be followed (see Figure 1). The reference optimal trajectory is then stabilized using two independent loops for controlling speed and lateral deviation. Currently the optimization phase, because of computational limitations, can be carried out only on a simplified model. To simulate the maneuver on the complete model, the optimal maneuver is computed on the simplified model and then the inner stabilizing loop is used to track the reference on the complete model. This approach is successfully applied in many conditions; but as the maneuvers become more extreme (with hard accelerations and high lean angles) the inner PID controllers cannot track the reference in a satisfactory way.

The scope of the present paper is that of improving the above internal controller by using a gain-scheduled roll-angle controller. Accordingly, the availability of an angle reference will be assumed. An accurate multi-body simulator of a sport motorbike [14] is employed to obtain a family of linear models that describe the roll dynamics for different working conditions, parametrized by longitudinal velocity and lateral acceleration. The models obtained via Jacobian linearization are

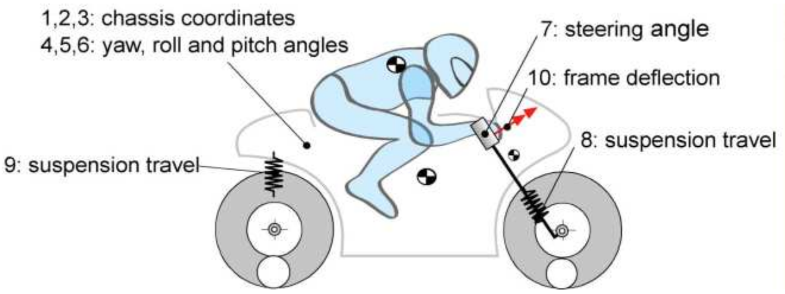

Figure 2. Simulator degrees of freedom.

helpful to derive several considerations from the control theory standpoint. The analysis of the linearised models shows that a fixed robust controller cannot guarantee satisfying performance; instead, Linear-Parameter-Varying (LPV) techniques yield a more performing controller.

The present work is structured as follows. In Section 2 the multi-body simulator and the linearization of the roll dynamics are described and analyzed. In Section 3 a background on LPV systems is provided and the algorithm employed in the synthesis is briefly recalled. In Section 4 two controllers are designed: a fixed-structure $\mathcal{H}_\infty$ controller and an LPV controller. Finally, in Section 5, the controllers are tested and validated using the full motorbike simulator. The paper ends with some conclusions and future work.

2 MOTORCYCLE MODELING

The mathematical modeling of two-wheeled vehicles is a challenging task and many models have been produced by many researchers, from simple analytical model to complex multibody simulators. Whereas simple analytical models are useful to understand key dynamical properties, the multi-body dynamic approach yields accurate models that are seldom too complex to be used for control system design.

The present study is based on the simulator developed by the Dinamoto group ([14]). The multi-body model is characterized by 10 degrees of freedom see Figure 2: chassis coordinates (3), chassis attitude (3), suspensions travel (2), frame deflection (1) and steering angle (1). The model also accounts for the deformation of the steering assembly. It is modeled as a lumped stiffness close to the steering handle (arrow in figure). The parameters of the simulator have been tuned with the data of a hypersport-class motorbike.

The multi-body simulator provides the possibility of analysing the system dynamics via Jacobian linearization. The model has been linearized around trim conditions characterized by constant longitudinal velocity and constant lateral acceleration (steady steady cornering); in particular the velocity has been varied from 25 to 60 m/s and the lateral acceleration from 0 to 12 m/s^2 with steps of 1m/s^2 (a total of 420 models are thus obtained). Figure 3 shows the pole map as the lateral acceleration and velocity vary. The figure shows several vibrating modes: front twist represents the structural mode associated to the steering handle stiffness, the hop modes are due to the radial deformation of the tires, the wobble, weave and the capsize are the out of plane modes typical of single track vehicles. Finally pitch and the bounce are due to the presence of the suspensions. As it is clear from figure the position of all the modes is strongly dependent on the longitudinal velocity and lateral acceleration.

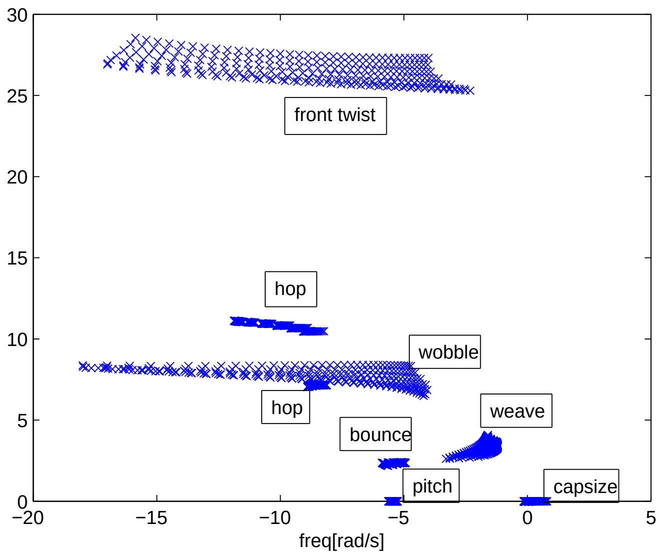

Figure 3. Vibrational modes for different longitudinal velocities and lateral accelerations.

The same technique can also be used to analyze the input-output dynamics of interest: namely from steering torque to roll angle. Figure 4 shows the bode diagrams of the magnitude obtained for different velocities and lateral accelerations. From figure the following considerations can be drawn:

- in straight running (upper plot) the out of plane dynamics and in plane dynamics are decoupled and thus the hop bounce and pitch modes are not visible. On the other hand, the weave, wobble and twist dynamics are clearly visible.
- The longitudinal velocity mainly influences the out-of-plane modes. In particular the weave frequency increases with the velocity while the wobble frequency decreases with an increase of velocity. The structural resonance is less damped at higher speed.
- During cornering the out-of-plane and in-plane dynamics are coupled. The weave, wobble and twist dynamics are still clearly visible but now they affect also the in-plane modes such as pitch and bounce.
- The lateral acceleration mainly influences the low frequency gain of the transfer function.

Also this analysis confirms the roll dynamics are strongly dependent on the trim conditions. In the next section it will be explained how this dependency can be taken into account in the control system design.

3 LPV SYSTEMS

In the previous section it was shown that the roll dynamics are dependent on the trim conditions. Modeling the variability of the system hence is very important. The Linear Parameter Varying

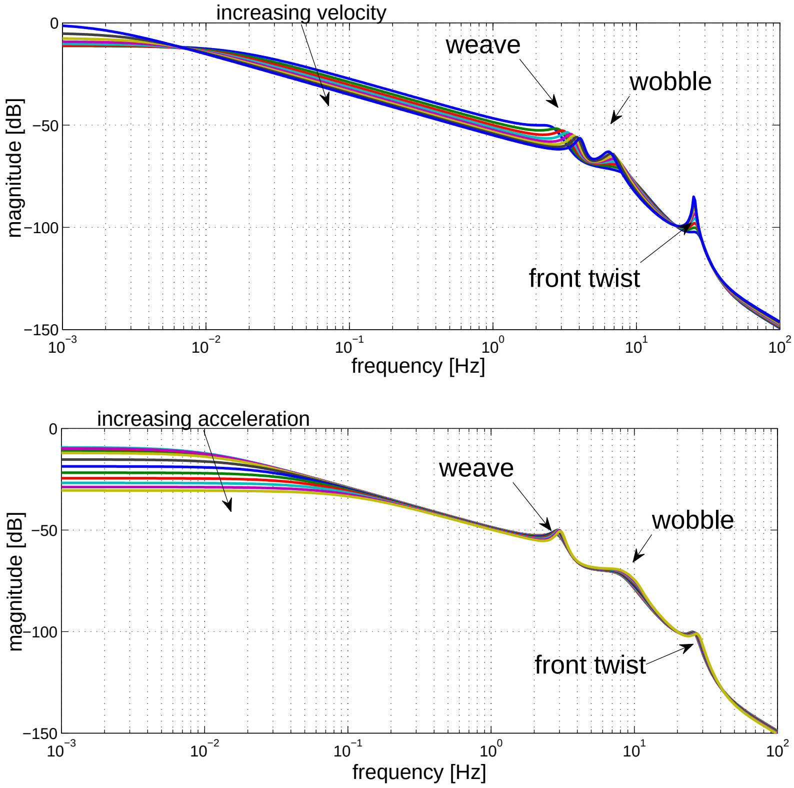

Figure 4. Bode diagrams of the linearized models. Upper plot: constant lateral acceleration $a_y = 0$ m/s^2, velocity $v = [25, 60]$m/s; Lower plot: constant velocity $v = 30$m/s velocity $a_y = [0, 12]$m/s^2.

(LPV) systems framework provides tools to both model this variability and to design scheduled controllers which adapt themselves to the varying dynamics and are guaranteed to retain stability and performance.

3.1 Main Theoretical Results

The basics of LPV control design are here briefly recalled (see [17] and reference cited therein for a detailed description of the LPV framework). LPV systems are linear systems, whose state-space descriptions are known functions of time-varying parameters $\rho \in \mathcal{P} \subset \mathcal{R}^s$. The parameters are assumed to be measurable in real-time and available to the controller. In this work it is moreover assumed that $|\dot{\rho}(t) \leq \nu|$, yielding a rate bounded problem [16].

After some technical assumptions [17] a generalized open loop LPV plant can be written as

$$\begin{bmatrix} \dot{x} \\ e_1 \\ e_2 \\ y \end{bmatrix} = \begin{bmatrix} A(\rho) & B_{11}(\rho) & B_{12}(\rho) & B_2(\rho) \\ C_{11}(\rho) & D_{1111}(\rho) & D_{1112}(\rho) & 0 \\ C_{12}(\rho) & D_{1121}(\rho) & D_{1122}(\rho) & I_{n_u} \\ C_2(\rho) & 0 & I_{n_y} & 0 \end{bmatrix} = \begin{bmatrix} x \\ d_1 \\ d_2 \\ u \end{bmatrix} \tag{1}$$

where, $d_1 \in \mathcal{R}^{n_{d1}}$, $d_2 \in \mathcal{R}^{n_{d2}}$, $e_1 \in \mathcal{R}^{n_{e1}}$, $e_2 \in \mathcal{R}^{n_{e2}}$ are partitions of the exogenous inputs and the disturbances. Given the system in (1) the final design goal is to find a parameter-depended output-feedback controller to stabilize the closed-loop LPV system and guarantee the induced L_2-norm of the closed-loop system less than γ. This can be done by resorting to the following theorem.

Theorem 3.1. ***LPV Control Synthesis [18]*** *Given a compact set $\mathcal{P}$, the performance level $\gamma > 0$, the LPV system (1) and a finite number of scalar, continuously differentiable functions $\{f_i\}_{i=1}^N$ and $\{g_i\}_{i=1}^N$, which will be referred as basis functions, with the parametrization*

$$X(\rho) = \sum_{i=1}^{N} f_i(\rho) X_i, \qquad Y(\rho) = \sum_{i=1}^{N} g_i(\rho) Y_i. \tag{2}$$

There exists a controller which pass the closed-loop stability and γ-performance test if there exist matrices $\{X_i\}_{i=1}^N$, $X_i \in S^{n\times n}$ and $\{Y_i\}_{i=1}^N$, $Y_i \in S^{n\times n}$ such that, for all $\rho(t) \in \mathcal{P}$, the following hold:

$$\begin{aligned} X(\rho) &> 0, & (3)\\ Y(\rho) &> 0, & (4) \end{aligned}$$

$$\begin{bmatrix} X(\rho) & I_n \\ I_n & Y(\rho) \end{bmatrix}, \tag{5}$$

$$\begin{bmatrix} \clubsuit & \sum_{i=1}^N f_i(\rho) X_i C_{11}^T(\rho) & \gamma^{-1}\hat{B}(\rho) \\ C_{11}(\rho)\sum_{i=1}^N f_i(\rho) X_i & -I_{n_{e_1}} & \gamma^{-1} D_{111\cdot}(\rho) \\ \gamma^{-1}\hat{B}^T(\rho) & \gamma^{-1} D_{111}^T(\rho) & -I_{n_d} \end{bmatrix} < 0 \tag{6}$$

$$\begin{bmatrix} \spadesuit & \sum_{i=1}^N g_i(\rho) Y_i B_{11}^T(\rho) & \gamma^{-1}\tilde{C}(\rho) \\ B_{11}^T(\rho)\sum_{i=1}^N g_i(\rho) Y_i & -I_{n_{d_1}} & \gamma^{-1} D_{11\cdot 1}(\rho) \\ \gamma^{-1}\tilde{C}^T(\rho) & \gamma^{-1} D_{11\cdot 1}^T(\rho) & -I_{n_d} \end{bmatrix} < 0 \tag{7}$$

where

$$\clubsuit = \sum_{i=1}^{N} f_i(\rho)\left(X_i \hat{A}^T(\rho) + \hat{A}(\rho) X_i\right) - \sum_{j=1}^{s} \pm \left(v_j \sum_{i=1}^{N} \frac{\partial f_i}{\partial \rho_j} X_i\right) - B_2(\rho) B_2^T(\rho)$$

$$\spadesuit = \sum_{i=1}^{N} g_i(\rho)\left(\tilde{A}^T(\rho) Y_i + Y_i \tilde{A}(\rho)\right) - \sum_{j=1}^{s} \pm \left(v_j \sum_{i=1}^{N} \frac{\partial g_i}{\partial \rho_j} Y_i\right) - C_2^T(\rho) C_2(\rho)$$

and

$$\begin{aligned} \hat{A} &= A - B_2 C_{12}, \quad B_1 = \begin{bmatrix} B_{11} & B_{12} \end{bmatrix}, \\ \tilde{A} &= A - B_{12} C_2, \quad C_1^T = \begin{bmatrix} C_{11}^T & C_{12}^T \end{bmatrix}. \end{aligned} \tag{8}$$

Theorem 3.1 provides a practical solution to the LPV synthesis problem. Once the functions $X(\rho)$ and $Y(\rho)$ are found, the admissible controller state space realization can be computed. Conditions (3)-(7) consist of $2^{s+1} + 1$ LMI's, which must hold for all $\rho(t) \in \mathcal{P}$. Notice that it is an infinite-dimension problem; many approaches have been developed to translate the infinite dimensional problem into a treatable problem (for example exploiting an affine representation of the parameter dependency or recurring to a linear fractional representation) here we will resort to parameter-space gridding. The synthesis equations provide a set of LPV controllers which guarantees local stability and performance near the grid points used in the design. Outside the grid vertexes, these controllers are linearly interpolated. In this setting the classical gridding trade-off arises: on one hand the complexity of the problem grows as the resolution of the grid; on the other hand, a tight grid guarantees a better description of the system and a smoother interpolation. This problem has been solved with the same technique introduced in [2].

3.2 LPV Models and LPV Controller Design for the Motorcycle

In Section 2 a family of 420 linearized models have been obtained. This 420-model gridding (say grid "A") is quite accurate, but it is too high-dimensional for the solution of the LMI problem associated with the synthesis of an LPV controller. To this end, a looser grid (say grid "B") of 36 models (3 values for the longitudinal velocity and 12 for the lateral acceleration) has been defined. One can think to assemble the two sets of linearized systems in an LPV system scheduled on the two exogenous inputs: longitudinal velocity and lateral acceleration. It is important to note that the obtained model is a quasi-LPV system because both velocity and slip are states of the system. While the velocity can be seen as a truly slowly-varying parameter, the approximation is less precise on the lateral acceleration.

To design the LPV controller, the problem (3)-(7) must be solved. The synthesis method is based on the two grids above described (the finer grid "A", and the looser grid "B"), as follows:

1. The LMI's described by conditions (3)-(7) are solved on grid B, and the weights of the 2N basis functions $X_{i}{}_{i=1}^{N}$,$Y_{i}{}_{i=1}^{N}$ are computed.
2. The basis functions are evaluated on grid A, so to obtain $X(\rho_k) = \sum_{i=1}^{N} f_i(\rho_k)X_i$, $Y(\rho_k) = \sum_{i=1}^{N} g_i(\rho_k)Y_i$ where ρ_k are the vertexes of grid A.
3. The open loop system is re-sampled (by local linearization of the non-linear simulator) on grid A, so to obtain $A(\rho_k)$,$B(\rho_k)$,$C(\rho_k)$,$D(\rho_k)$.
4. The matrices $A(\rho_k)$,$B(\rho_k)$,$C(\rho_k)$,$D(\rho_k)$, $X(\rho_k)$ and $Y(\rho_k)$ are used to synthesize the controller on grid A.

This method improves the smoothness of the controller interpolation without increasing the number of LMI's. Another problem common in the synthesis of LPV controllers is the presence of high frequency poles in the controller. In order to alleviate this problem, two rate-bounded LPV controllers are synthesized. The first formulates the standard LPV control algorithm; the second uses the first solution and includes an additional constraint on the closed-loop system poles at the grid points, as described in [17, 16].

In conclusion, LPV methods provide a systematic design of gain-scheduled controllers that includes performance and robustness objectives in the design process.

4 ROLL ANGLE CONTROL

In this section, the design of the roll angle controller is discussed. Two control strategies are presented. First, a robust fixed structure $\mathcal{H}_\infty$ controller is designed, then a scheduled controller is designed and it is shown that, by adapting to the changing conditions of the system it can achieve better performances.

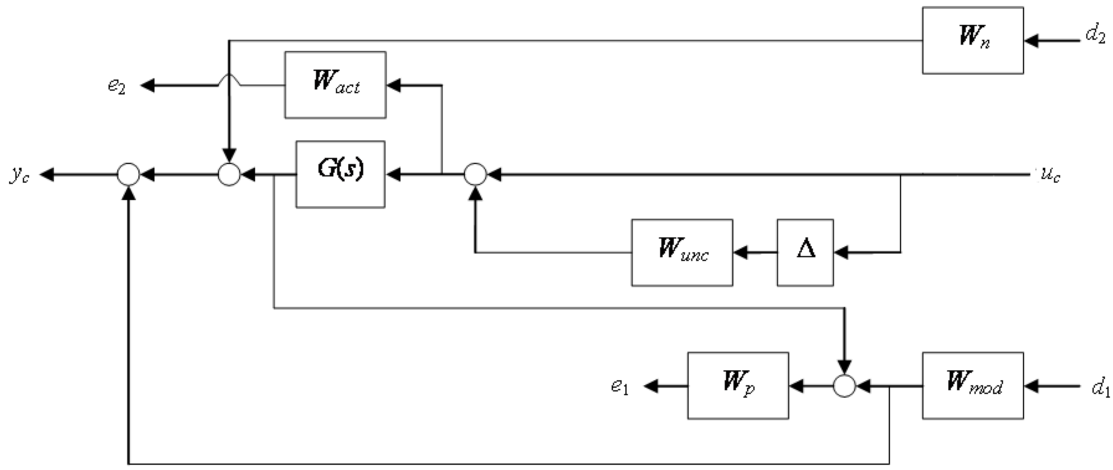

Figure 5. General structure of the system, for the design of the $\mathcal{H}_\infty$ controller.

4.1 Robust Fixed-Structure Design

The problem considered herein is the regulation of the lean angle of the vehicle using the steering torque as control variable. Although the system under study is a Single-Input-Single-Output (SISO) system, it has been found to be advantageous to add a second output: roll angle rate.

One of the possible ways to address the variation of the plant dynamics is through robust control. The underlining idea is that of designing a single controller that is stable for all the possible conditions. This objective can be achieved via the $\mathcal{H}_\infty$ framework [19]. Figure 5 shows the complete block scheme used to design the controller. Each element of the block scheme is now briefly discussed:

- $W_{mod}(s)$ represents the set point filter with $d_1 = [\varphi_{ref}, \dot{\varphi}_{ref}]$. It is a simple diagonal 1st-order linear model with one pole at 20 Hz and no zeros.
- The tracking error is weighted by means of the weighting filter

$$W_p(s) = \begin{bmatrix} \frac{500}{s/0.0063+1} & 0 \\ 0 & \frac{s+1}{(s+2\pi)(s+6\pi)} \end{bmatrix}.$$

  Notice that the low-frequency matching error is penalized, so to guarantee a small DC-error.
- The model of the output disturbance has been simply chosen as $W_n(s) = 0.002$. This corresponds to assuming white-noise output disturbance; notice that although incorporating this weighting function is not necessary (the final goal of the controller is to run in a simulator), accounting for it may render the controller more robust to numerical errors.
- The weighting filter $W_{act}(s)$ is used to limit the bandwidth of the controlled system. In the robust controller it has been chosen to penalize the bandwidth above 2 Hz.

- $G(s)$ models the nominal plant. In the $\mathcal{H}_\infty$ framework the variation of the system is accounted for as uncertainty. Specifically, the variability of the dynamics is modeled as a multiplicative uncertainty. This choice yields the following family of perturbed plants:

$$\mathcal{M}_{Gp} = \{G_p(s) = (1 + \Delta_m(s))G(s) : |\Delta(j\omega)| \leq W_{unc}(j\omega)\forall\omega\}.$$

where the nominal plant $G(s)$ is obtained at a velocity of 50m/s and a lateral acceleration of 3 m/s^2.

Figure 6 shows the uncertainty Δ_m and weighting function W_{unc}. Notice that the uncertainty peaks at 2 Hz (the weave frequency). This effectively limits the achievable bandwidth around that frequency.

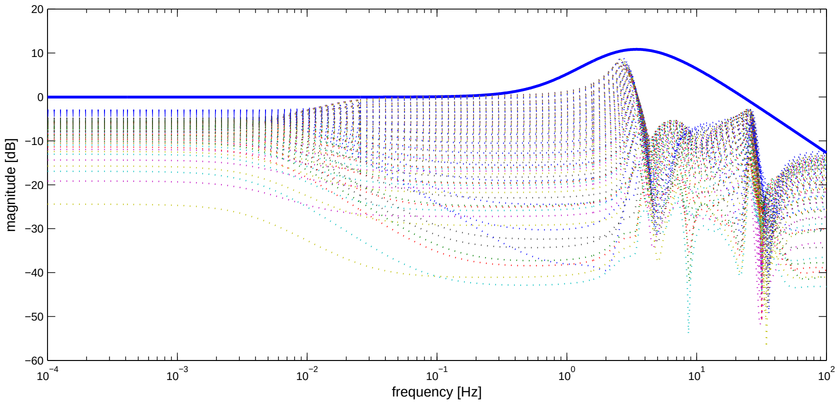

Figure 6. Uncertainty Δ_m and weighting function W_{unc}.

From the above set-up, the transfer function of the controller can be easily computed using one's preferred robust control toolbox. Figure 7 plots several closed-loop step responses for different longitudinal velocities and lateral accelerations for the obtained controller. The $\mathcal{H}_\infty$ fixed-structure

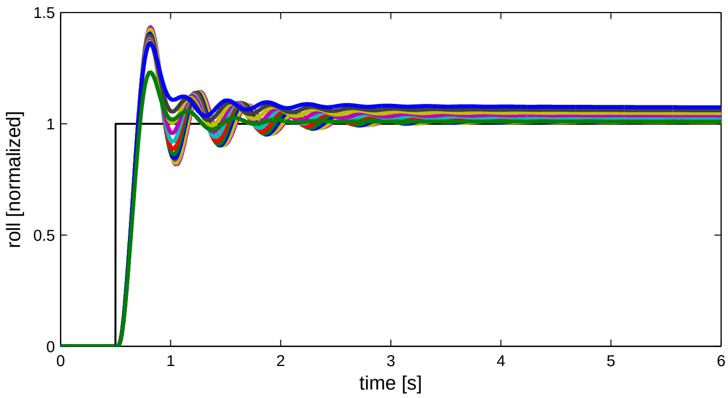

Figure 7. Closed-loop step responses for several longitudinal velocities and lateral accelerations.

controller provides a stable control system for all linearized systems, but the performances are not satisfying. The undamped weave oscillation is clearly visible in the response and also the steady state value depends on the linearization condition.

4.2 Scheduled LPV Design

A way to solve the problem of large performance variations is to change the controller structure, by allowing it to adapt to the plant dynamics. The interconnection scheme shown in Figure 5 can be re-used by simply removing the weighting function W_{unc}. The removal of the uncertainty due to the changing variation allows for a more *aggressive* tuning of the controller. This can be done by changing the weighting filter $W_{act}(s)$ that now penalizes the use of the actuator above 50 Hz.

The conservativeness of the LPV design is reduced by using the rate-bound formulation: the rate bound on the velocity is ± 10 m/s^2 and the rate bound on the lateral acceleration is $\pm 20 m/s^3$. The basis functions used to approximate the infinite dimensional LPV problem are:

$$X(v, \lambda) = X_0 + vX_1 + a_y X_2$$
$$Y(v, \lambda) = Y_0 + vY_1 + a_y Y_2$$

The computation effort to solve the problem is non-negligible (it takes about 1h on a standard PC). Moreover it should pointed out that, regardless of the second set of bounded LMI, the obtained controller still have very high frequency (pole up to 10^5 Hz). It is believed that these poles are not needed from a control point of view but are an artifact of the choice of weights. Further investigation is being carried out to address this particular issue.

The performances of the LPV controller are displayed in Figure 8 for constant value of the parameters. As expected, the closed-loop results are better than in the fixed structure controller. The bandwidth of the close-loop system is almost invariant and the weave mode is better damped.

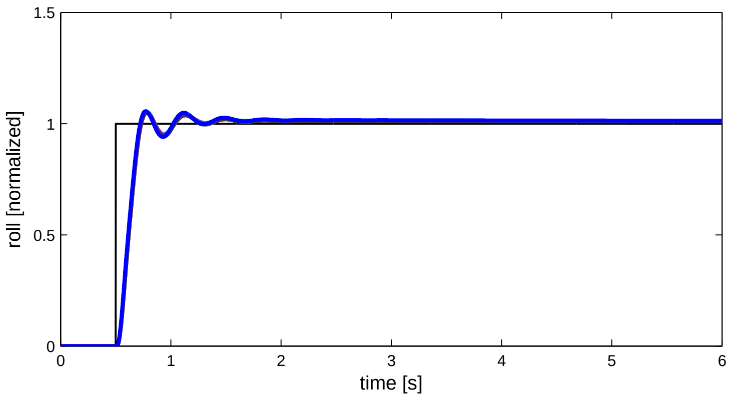

Figure 8. Closed-loop step responses of the LPV controlled system for several different longitudinal velocities and lateral accelerations.

5 SIMULATION RESULTS

The LPV motorcycle model derived in Section 2 is an approximation of a nonlinear system; since the synthesis techniques employed guarantee stability and performance only for the LPV model, a validation of the controller on a more realistic simulation is needed. In this section the complete multi body simulator is used to validate the two controllers. In particular two conditions will be discussed: a gentle cornering maneuver executed at constant speed and a more aggressive cornering.

5.1 Implementation Issues

It has been pointed out that regardless of the formulation of the second, bounded, LMI problem the synthesis yields a controller with high frequency dynamics. This generates two main problems: long simulation times and the tendency to exhibit numerical instability when fast variations of the lateral acceleration are involved. Efficient simulation is currently out of scope of this investigation; on the other hand numerical instability may represent a problem. The numerical stability of the simulation has been improved by avoiding the direct feedback of the lateral acceleration into the controller as a scheduling variable. Instead, the steady state motorcycle model (see Figure 9) has been used to generate the expected lateral acceleration from the lean angle reference. The model

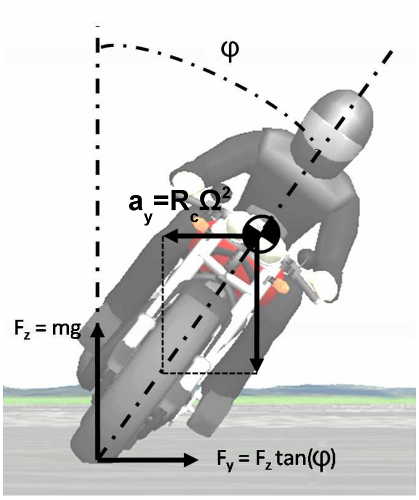

Figure 9. Steady turning: roll angle and lateral acceleration of the motorcycle equipped with zero thickness tires.

assumes steady state cornering with zero thickness tires. In these conditions the lateral acceleration is immediately derived from the roll angle that $\varphi = g\,\arctan(a_y)$. In all the following simulations the scheduling lateral acceleration will be computed using the steady state lateral acceleration.

5.2 Discussion

Once the numerical issues have been solved the performance of the proposed controllers can be evaluated. Figure 10 shows the results obtained for the gentle maneuver. The maneuver consists of a constant velocity (30 m/s) cornering reaching a maximum roll angle of 20°. A smooth reference lean angle is generated as a sigmoid with a rise time of 1s. In these conditions both the proposed controllers behave reasonably well, nevertheless some considerations can be drawn:

- the roll angle of the fixed $\mathcal{H}_\infty$ controller shows the same rise time as the LPV controller; the differences are seen at the end of the sigmoid where a 1° overshoot is visible; further the fixed structure controller exhibits oscillations in the constant lean angle phase.
- Both controllers yield a rather smooth steering torque, the main difference being the oscillations visible in the $\mathcal{H}_\infty$ controller.
- The oscillations are visible especially in the lateral acceleration. Note that in this case, the lateral acceleration reaches a value that is close to the value used for the nominal model.

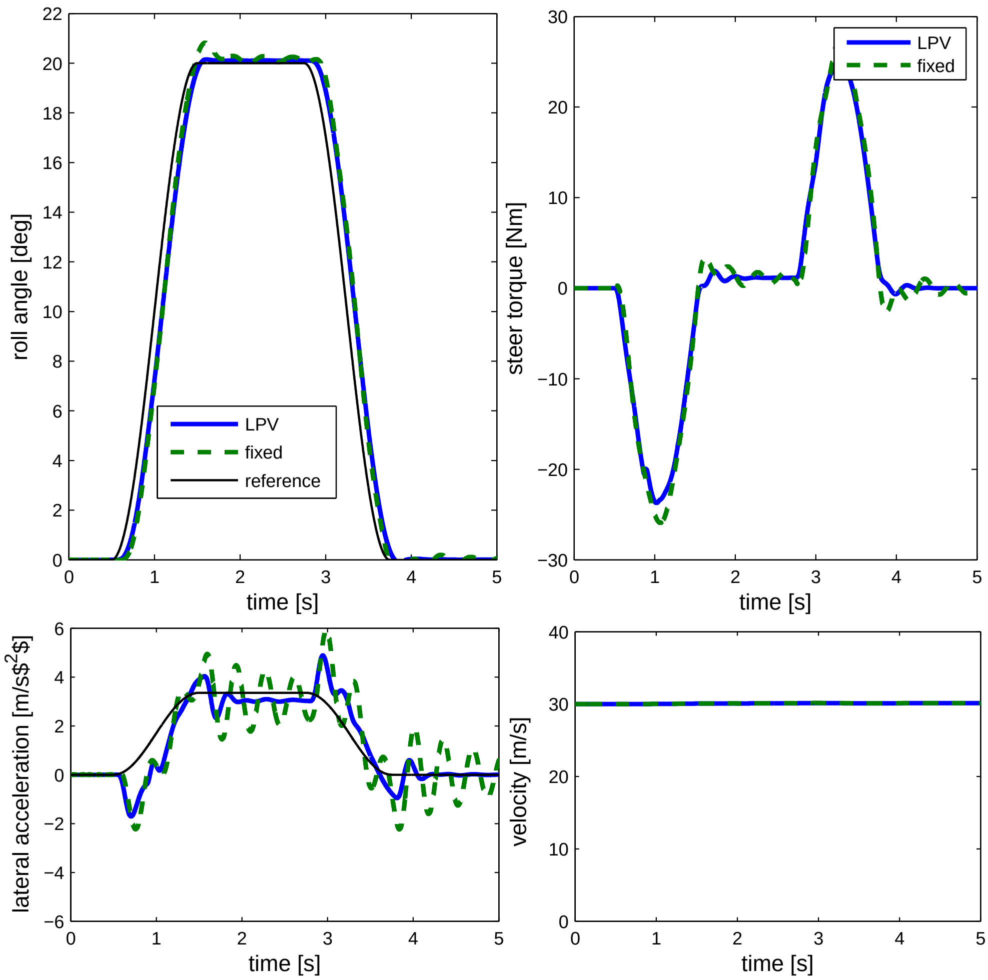

Figure 10. Comparison between the $\mathcal{H}_\infty$ controller and the LPV controller for a gentle maneuver: roll angle, steering torque, lateral acceleration and longitudinal velocity.

- It is also interesting to note that the generated a_y used for scheduling is quite accurate in the constant lean phase, while it is less accurate in the transients. In particular notice that it fails to model the non minimum phase behavior typical of motorcycles. Nevertheless the LPV controller can still deliver satisfying results.

The second maneuver consists of a more extreme cornering maneuver, the maximum lean angle is now 60° (with the same rise time as before); further, once the reference lean angle starts to decrease the throttle is opened to accelerate out of the corner. The results, with the comparison between the two controllers, are shown in Figure 11. The difference between the gentle maneuver is immediately clear. The fixed controller cannot successfully negotiate the corner. As soon as the throttle is opened the motorcycle falls. Further notice the damped oscillations in the lateral acceleration for the LPV controller; they can be explained by recalling that the LPV model has been generated by compositions of linearized models in steady state cornering; when the motorcycle accelerates the front tire unloads (in this specific case the final dynamic load is half the static load), this has an effect on the wobble mode and thus the controller cannot perfectly damp the

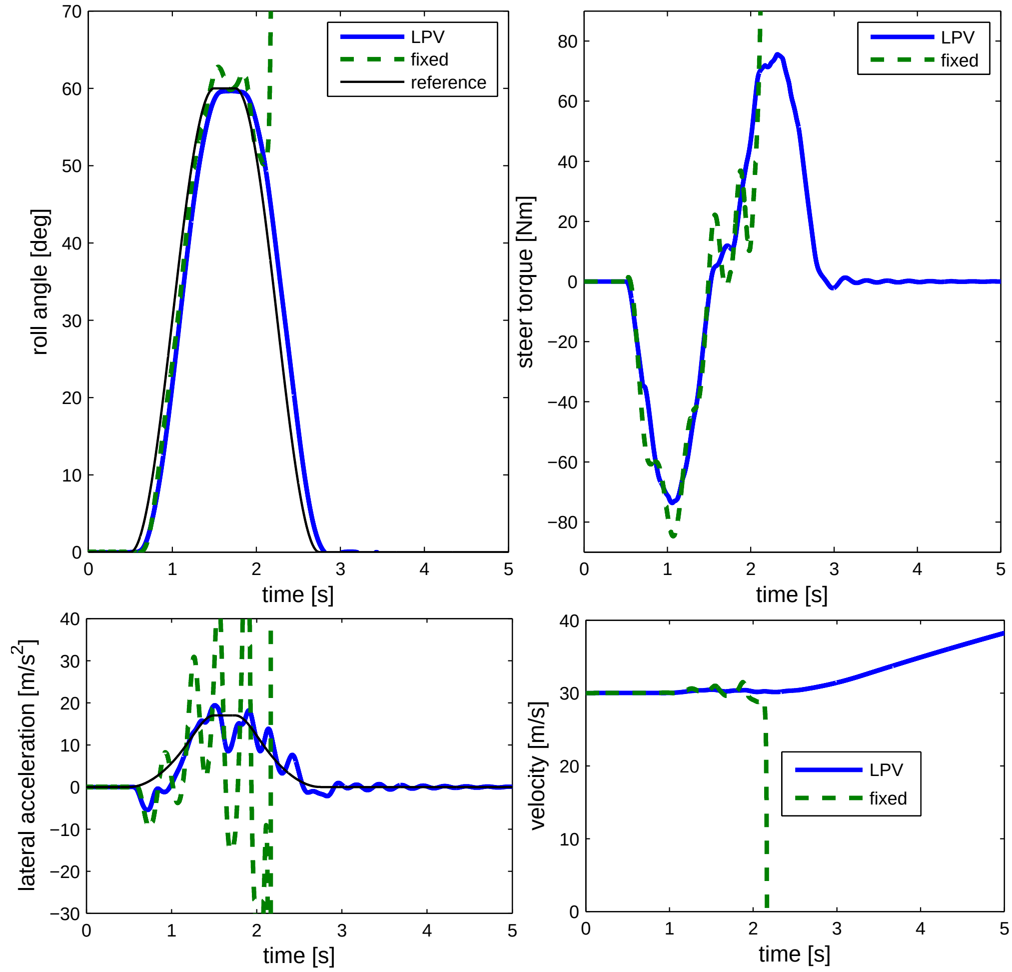

Figure 11. Comparison between the $\mathcal{H}_\infty$ controller and the LPV controller for an aggressive cornering maneuver: roll angle, steering torque, lateral acceleration and longitudinal velocity.

oscillation. Although the dependency of the wobble mode on the front tire load is not accounted for, the proposed controller is rather robust in that respect.

6 CONCLUSIONS

In the present work the problem of designing a roll angle controller for simulation of sport motorcycles has been addressed. A multi-body nonlinear simulator has been employed to obtain a Linear Parameter-Varying model of the open loop dynamics: the dynamics is strongly dependent on the vehicle longitudinal velocity and lateral acceleration. Two controllers were designed: a fixed structure controller and an LPV controller. The LPV controller can better adapt to the varying dynamics and therefore it achieves better performance. The proposed controllers were validated on the multi-body simulator: it was shown that the fixed and LPV controller are equivalent in case of gentle maneuver; if the simulated motorcycle is pushed toward its limit in terms of lean angle and longitudinal acceleration, the LPV controller shows its advantages.

Although the validity of the LPV controller have been validated further work need to be done in order to obtain faster simulation time and a more numerically robust simulation. Further research is being carried out toward a MIMO controller to coordinately control longitudinal velocity and roll angle (see Figure 1).

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