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Assessing slip of a rolling disc and the implementation of a tyre model in the benchmark bicycle

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Abstract

For the development of a real time bicycle/motorcycle model a compact formulation of the dynamic behaviour is required. The modelling environment MatLab SimMechanics was posed as a prerequisite.

For the development of the tyre model the wheel was considered as a disc with knife edge contact to the ground. A tyre model typically uses slip quantities as the input and calculates forces and moments as outputs. Longitudinal- and lateral slip are calculated as the components of a normalized slip velocity in the contact point. The location of this contact point can be denoted with a position vector **r** pointing from the wheel disc centre to the contact point.

The assessment of the slip quantities has been based entirely on the vector calculation presented by Pacejka in [1]; the contact point can be found in radial direction $\mathbf{e}_r$ at a scalar distance r from the wheel centre. Here $\mathbf{e}_r$ is recursively defined as being orthogonal to the axial $\mathbf{e}_s$ and longitudinal direction $\mathbf{e}_l$, where the longitudinal direction is the intersection of the road plane and wheel plane, thus the vector orthogonal to road normal **n** and axle direction $\mathbf{e}_s$. See Figure 1

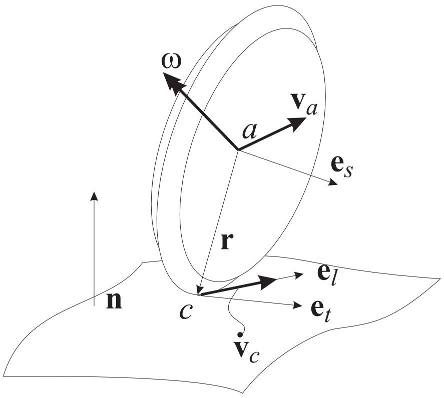

Figure 1. The wheel disc with contact point location **r**

In SimMechanics a single rolling disc has been modelled. With the motion of the centre of the wheel disc and the vector **r** available, the derivation of the velocity of the material point s on the wheel, momentarily in the contact point c, is straightforward: $\mathbf{v}_s = \mathbf{v}_a + \boldsymbol{\omega} \times \mathbf{r}$. This velocity of the point s needs to be normalized with the wheel speed v_x, to obtain the slip quantities α and κ that are the inputs for the tyre model.

$$\lambda = \frac{\mathbf{v}_s}{v_x}, \quad \tan\alpha = -\lambda_y, \quad \kappa = -\lambda_x$$

A simple linear tyre model defines the dissipative tyre contact forces, proportional to the established slip quantities. However normalizing slip with the wheel-centre longitudinal velocity $v_x \rightarrow v_{a,x}$ is practical, but incorrect. We show that the singularity caused by dividing by zero axle speed, poses difficulties for the numerical solvers when simulating the wheel as an Euler disc. The correct velocity to use in the normalization of slip is the propagation speed of the contact point $v_{c,x}$, which comprises $v_{a,x}$ and a term with $\dot{\mathbf{r}}$. Since $\mathbf{r}$ is the result of sequential cross products, its time derivation is awkward, yet possible.

Using the propagation speed in the denominator, the single wheel behaves as an Euler disc in simulation. Due to the finite slip stiffness assumed in the model the quantative behaviour deviates somewhat form the Euler disc with a non-holonomic constraint in the contact. The wheel model is further elaborated by implementing first order relaxation equations that enable more realistic transient tyre behaviour. The introduction of the relaxation equation is also known to facilitate simulating at zero velocity.

Two of these wheel-disc models were assembled with a rear-frame and a front-fork body in SimMechanics to build a so-called Whipple bicycle model [2]. With the bicycle model parameterised according to the benchmark, numerous time simulations have been carried out. The roll angular velocity response has been used to curve-fit a standard exponential response. The eigenvalue can be found as from the (complex) exponent of the optimally fitted time response. The eigenvalues obtained for many velocities can be visualized in the so-called root-loci plot. The figure shows excellent correspondence between the benchmark Whipple bicycle with kinematic rolling constraints, and our bicycle with linear transient tyre models.

The difference with kinematic rolling will be exaggerated by reducing the slip stiffness in our model. Also the effect of transient, compliant tyre behaviour will be studied by simulating the model using increased relaxation lengths. The conclusion is that the model can be parameterised to match the kinematic rolling, and can be used to as a starting point to implement more realistic (non-linear) tyre behaviour when required for specific (future) applications of this bicycle model.

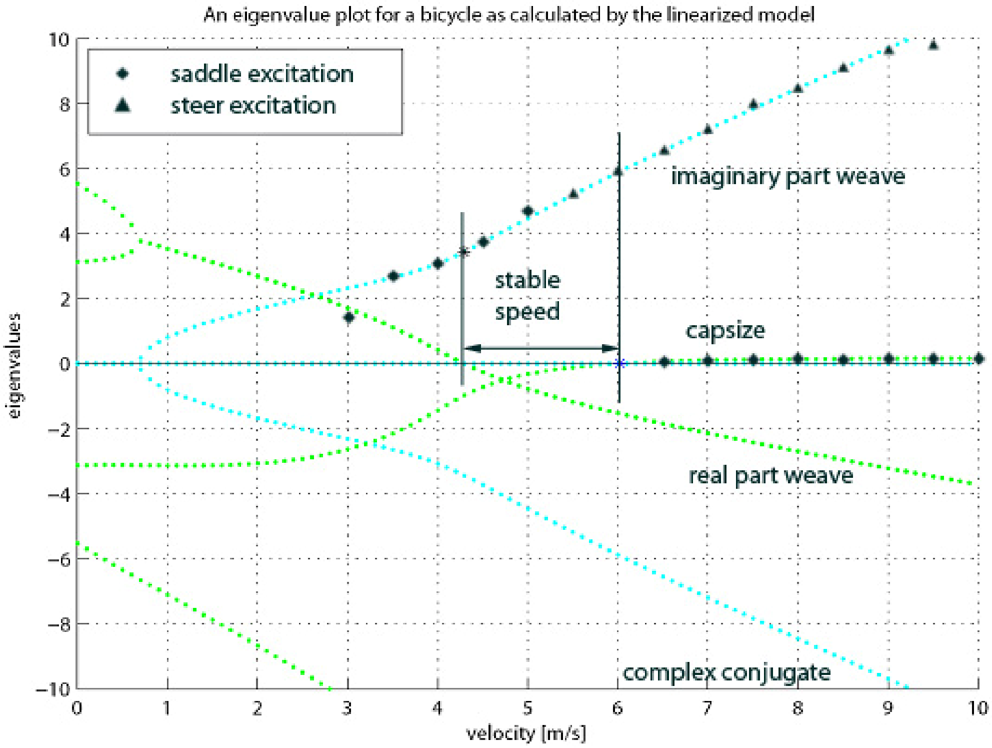

Figure 2. The root loci of the benchmark bicycle and a multi-body model of a Whipple bicycle with linear transient tyre models. The multi-body model has been excited on the saddle, eigenvalue marked with ♦, and on the handlebar, marked with ▲.

References

[1] H. B. Pacejka, *Tyre and Vehicle Dynamics*, Butterworth and Heinemann, Oxford, 2002.

[2] J.P Meijaard, Jim M Papadopoulos, Andy Ruina and A.L Schwab, Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review, *Proceedings of the Royal Society A.* **463** (2007), pp. 1955-1982