Bicycle and Motorcycle Dynamics 2010
Symposium on the Dynamics and Control of Single Track Vehicles,
20 - 22 October 2010, Delft, The Netherlands

Comparison of a bicycle steady-state turning model to experimental data

S. M. Cain
PhD Candidate, Biomedical Engineering
University of Michigan
Ann Arbor, MI, USA
e-mail: smcain@umich.edu

N. C. Perkins
Professor, Mechanical Engineering, University of Michigan

Abstract

The design of the modern bicycle is the result of almost 200 years of trial and error. Recent work has helped us to understand the stability of a bicycle and has shown that the current bicycle configuration could be made more stable with relatively small adjustments to standard bicycle geometry [1]. However, stability is not the only characteristic that a human rider desires; a bicycle also needs to be maneuverable.

To examine maneuverability, we begin with the steady-state handing of a vehicle. In particular, one can examine the gain or sensitivity of a vehicle output (lateral acceleration) to a vehicle input (steering torque), which has important implications for human control [2, 3]. The goal of this study is to develop a parallel steady-state handling model of a bicycle and rider and an instrumented bicycle to test model fidelity.

We develop a steady-state handling model for a bicycle with rigid rider making a turn of constant radius with constant speed, similar to the linear steady-state model developed by Pacejka [4]. Relative to the dominant forces (lateral tire force and weight), we neglect air drag, longitudinal tire forces, and vertical tire moments. We assume the turn radius is much larger than the bicycle wheelbase and assume small steer and roll angles (≤ 5°). The equations governing steady-state handling are derived from Newton's law in the lateral direction and moment equilibrium about the vertical axis. The tires obey the linear elastic tire model

$$F_{yi} = C_{F\alpha i}\alpha_i + C_{F\gamma i}\gamma_i \tag{1}$$

where i is an index denoting the tire (front or rear), F_y is the lateral force on the tire, $C_{F\alpha}$ is the slip or cornering stiffness, α is the side slip angle, $C_{F\gamma}$ is the camber stiffness, and γ is the camber angle. We use three sets of tire stiffness values: those from Roland [5] and Sharp [6] for elastic tires, and those for an idealized tire model ($C_{F\alpha} = \infty$, $C_{F\gamma} = 0$). The gain or sensitivity is a function of velocity as given by

$$a_y/T_\delta = u^2/(Pu^2 + Q) \tag{2}$$

where a_y is the lateral acceleration of the bicycle and rider center of mass, T_δ is the steering torque, u is the bicycle/rider forward speed, and P and Q are constants that are determined by the tire stiffness values, head tube angle, trail, wheelbase, and weight distribution of the bicycle/rider system for our instrumented bicycle. The instrumented bicycle is a standard geometry mountain bike (head angle = 72°, trail = 58mm, wheelbase = 1.047m) fitted with 1.95" x 26" slick tires (Figure 1A). The instrumented bicycle allows us to measure steer torque (load cell), steer angle (optical encoder), bicycle lean angle and rider lean angle (3-axis accelerometer), and bicycle speed (magnetic reed switch).

The experiment considered two subjects following two level circular paths (radii 13.7 and 18.3m). The measured torque and velocity data during steady-state turning was broken into 5 second blocks and averaged to create discrete data points (Figure 1B). The steady-state turning model predicts a negative relationship between lateral acceleration and steer torque, which agrees with the experimental results (Figure 1B). All models explain the variance in the experimental data equally well (Table 1, r^2), while the idealized tire model minimizes the error (Table 1, SSE). An experimentally determined model follows by fitting Eq. (1) to the experimental data to find P and Q. Q is the only significant parameter of the model (α = 0.05), and it is also significantly different from the Q of the other models (α = 0.05).

Table 1. Comparison of different steady-state models and the fit to experimental data

Model	***P***	***Q***	**Sum of squares due to error (SSE)**	**r^2**
Idealized tire, subject 1	0	-5.806	87.73	0.726
Roland [5] tire, subject 1	0.0084	-5.779	101.23	0.726
Sharp [6] tire, subject 1	0	-5.677	95.80	0.726
Experimental data fit, subject 1	0.006707	-7.473	52.74	0.726

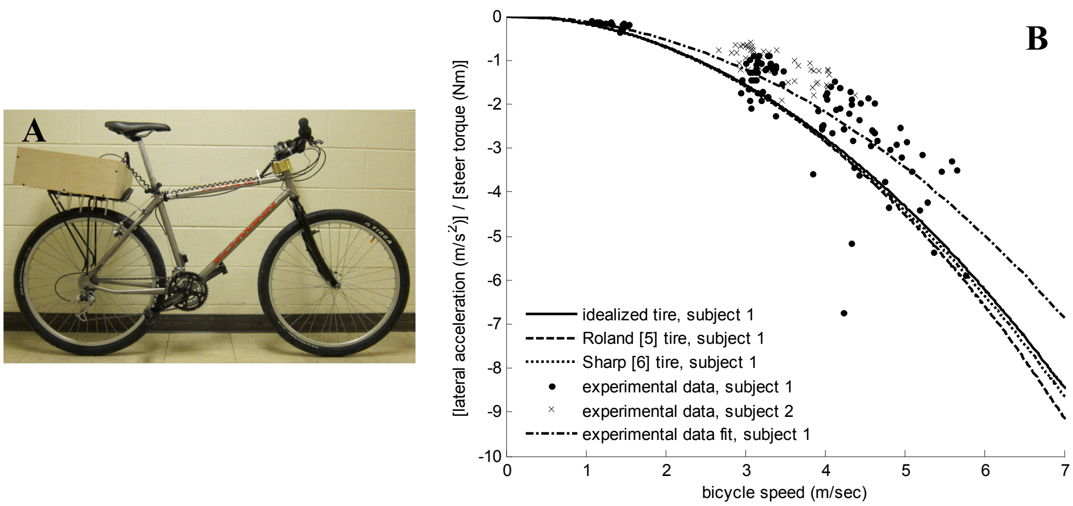

Figure 1. (A) The instrumented bicycle **(B)** The ratio of lateral acceleration over steer torque versus speed; the results of the model for three sets of tire parameters are plotted with the experimental data of 2 subjects and a best fit to the experimental data of subject 1.

Via the instrumented bicycle, we show that a steady-state turning model provides a good fit to experimental data. The model indicates that tire parameters do not have a pronounced effect on the control strategy that must be employed by a human rider.

References

[1] J. Moore and M. Hubbard. "Parametric study of bicycle stability", in M. Estivalet and P. Brisson (eds), *The Engineering of Sport 7* **2** (2008), pp. 311-318.

[2] D. H. Weir. "Motorcycle handling dynamics and rider control and the effect of design configuration on response and performance", PhD thesis, UCLA, CA, 1972.

[3] A. J. R. Doyle. "The essential human contribution to bicycle riding", in J. Patrick and K. D. Ducan (eds), *Training, Human Decision Making and Control*, Elsevier Science Publishers B. V., North-Holland, 1988, pp. 351-370.

[4] H. B. Pacejka, *Tyre and Vehicle Dynamics,* Butterworth and Heinemann, Oxford, 2006.

[5] R. D. Roland. "Computer simulation of bicycle dynamics", in J. L. Bleustein (ed), *Mechanics and Sport*, ASME, New York, 1973, pp. 35-83.

[6] R. S. Sharp. "On the stability and control of the bicycle", *Applied Mechanics Reviews* **61** 060803 (2008).