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A dynamic inversion approach to motorcycle trajectory exploration

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Abstract

The dynamics of two-wheeled vehicles is extremely rich. It exhibits, to cite the most important features, unstable dynamics, underactuation, mechanical symmetries, countersteering ("steer left to turn right"), a wide range of operating conditions, and speed dependent transitions from instability to stability regions. From a control theory point of view, the maneuvering control problem for these mechanical systems still poses an interesting number of challenges. Our interest, in this context, is the the development of control strategies for the exploration of two-wheeled vehicle dynamics in virtual prototyping studies for high performance motorcycles [1, 2].

Theoretical investigation on simplified motorcycle models [3, 4] suggests that it seems possible to *uniquely* parameterize any *upright roll* motorcycle trajectory of *infinite time extent* (i.e., $t \in \{-\infty, +\infty\}$) by means of the corresponding flatland trajectory traversed by the rear wheel contact point on the ground. Inspired by this idea, in [5] it was shown that the trajectories of a nonholonomic motorcycle can be parametrized by the trajectories of a nonholonomic car. Given a time indexed plane curve (including appropriate derivatives), a state-control trajectory of a nonholonomic motorcycle that (approximately) implements a desired plane trajectory was obtained.

In this paper, we introduce a *rigid motorcycle model* which captures many important aspects of real motorcycle dynamics including sliding and load transfer. This model is used to demonstrate a dynamic inversion procedure which maps a desired *flatland* trajectory into a corresponding (state-control) trajectory for the rigid motorcycle model. We assume that the rider is firmly attached to the main body of the vehicle, leaving the discussion on the important control and configuration effects offered by rider motion for future investigation.

In [5], the dynamic inversion was obtained *embedding* the motorcycle dynamics into an extended control system containing a sufficient number of additional *artificial controls* to make the system trivially controllable. We follow the same approach here, obtaining a fully actuated mechanical system starting from the underactuated rigid motorcycle model.

Due to the trivial controllability of the extended control system, we can can make it follow any desired velocity-curvature profile. Subsequently, the effect of artificial controls can be optimized away, obtaining a suitable motorcycle trajectory consistent with the requirements.

To solve the optimal control problem which is required to remove the effect of the artificial controls, we use the *projection operator approach* detailed in [6]. This optimization approach exhibits second order convergence rate in a neighborhood of the solution, being a generalization of Newton method in the infinite dimensional setting. We remark that the dynamic inversion procedure we propose in this work is suitable to be extended to more detailed motorcycle models, where, e.g., steering torque input, nontrivial steering geometry, and rider motion are considered.

In the paper we first introduce the *sliding plane motorcycle* (SPM) model, a rigid motorcycle model whose dynamics will be inverted using optimization. Then, the mechanical symmetries of the SPM model are described, and its dynamics are formulated in body coordinates. The *equilibrium manifold* for the SPM model, whose existence derives from the presence of mechanical symmetries, is analysed. We then present the (approximate) dynamic inversion strategy, that we term *lifting*, that "lifts" a flatland trajectory to a full state trajectory for the SPM model, using an optimization strategy. Effectiveness of the method is shown by means of numerical simulations. The paper will also include some discussion on the use of the lifted trajectory to design a maneuver regulation controller (virtual rider) to make a multibody motorcycle model follow a desired curvature-velocity profile [2].

References

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