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A bicycle model for education in machine dynamics and real-time interactive simulation

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Abstract

The bicycle model presented in this paper was originally developed for educational purposes at the University of Seville for teaching kinematics and dynamics of machines at the master level. The model is shown in Figure 1. The system is kinematically described by the following set of coordinates that can be identified in the figure:

$$\mathbf{p} = [x_C \quad y_C \quad \varphi \quad \theta \quad \psi \quad \beta \quad \gamma \quad \varepsilon \quad \xi]^T \tag{1}$$

where the angle ξ that is used to locate the contact point on the front wheel is a non-generalized coordinate.

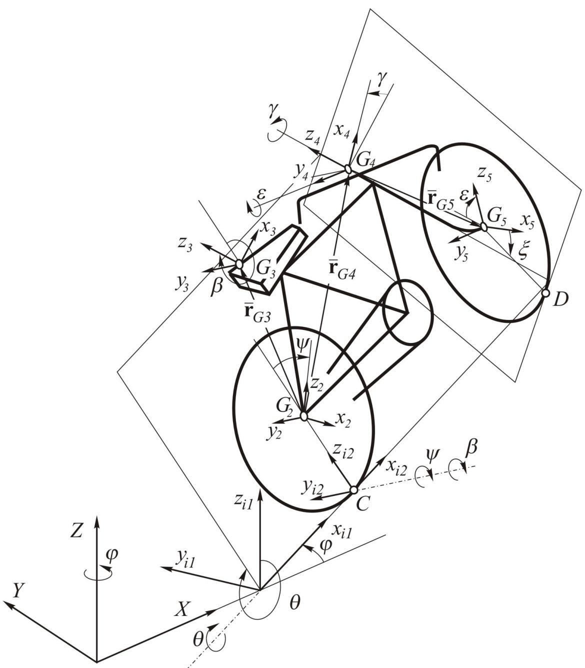

Figure 1. Biclycle model and coordinates description

The set of coordinates $\mathbf{p}$ is subjected to contact constraints $\mathbf{C}^{con}(\mathbf{p})$ (scleronomic), rolling without sliding constraints $\mathbf{C}^{rol}(\mathbf{p},\dot{\mathbf{p}})$ (non-holonomic) and mobility constraint $\mathbf{C}^{mov}(\mathbf{p},t)$

(rehonomic). This bicycle model can be used to explain the differential-algebraic (DAE) nature of the equations of motion that appear in multibody dynamics and different methods that can be used to deal with them. These equations that obtained in this investigation using symbolical computations take the form:

$$\begin{aligned} &\mathbf{M}(\mathbf{p})\ddot{\mathbf{p}} + \mathbf{D}^T\boldsymbol{\lambda} = \mathbf{Q}_v(\mathbf{p},\dot{\mathbf{p}}) + \mathbf{Q}_{grav}(\mathbf{p}) + \mathbf{Q}_{ext} \\ &\mathbf{C}(\mathbf{p},\dot{\mathbf{p}},t) = \mathbf{0} \end{aligned} \tag{2}$$

The equations of motion in DAE form given in Equation (2) are transformed to a minimal set of ordinary differential equations (ODE) using the generalized coordinate partition method [3]. The lean angle θ and the steer angle γ are selected as independent coordinates and the new equations take the form:

$$\mathbf{M}_i\ddot{\mathbf{p}}_i = \mathbf{Q}_i \tag{3}$$

Equations (3) are linearized about the vertical position and eigenvalue analysis is carried out for a range of forward velocities. The results are shown in Figure 2. This continuation diagram is compared to the one given in Reference [2] showing good agreement and it has been experimentally verified.

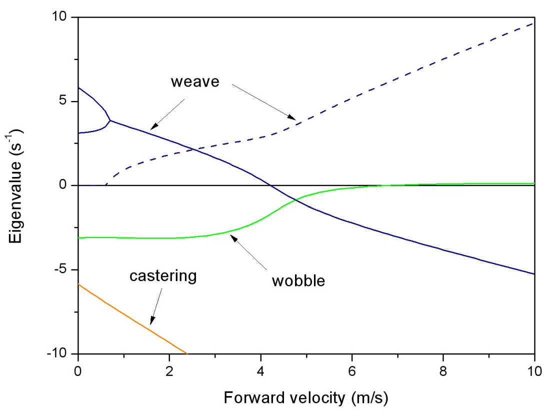

Figure 2. Continuation diagram of the bicycle stability

The symbolically obtained equations of motion of the bicycle are well suited for interactive real-time simulation of the bicycle riding. The simulator developed in this investigation uses steering wheels, joysticks and inertial sensors used in video games for virtual riding of the bicycle. The simulator also includes simple 3D visualization. Different strategies for virtual riding of the bicycle are tested. The inputs to the system may include the steering angle γ, a steering torque M_γ^x in the forward direction, a steering torque M_γ^z in the direction of the front fork axis of rotation and an upper body lean angle. The bicycle motion obtained with the simulator seems very realistic however path following control requires further development. The simulator is a nice application of control of underactuated systems.

References

[1] R.A. Wehage and E.J. Haug, "Generalized coordinate partitioning for dimension reduction in analysis of constrained dynamic systems", *J. Mech. Des.* **134** (1982), pp. 247–255.

[2] J.P. Meijaard, J.M. Papadopoulos, A. Ruina, and A.L. Schwab, "Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review", *Proceedings of the Royal Society A*, **463** (2007), pp. 1955-1982.