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Modeling Mechanical Optimization in Competitive Cycling

P. Cangley
Chelsea School
University of Brighton
Gaudick Road, Eastbourne, United Kingdom
e-mail: p.cangley@brighton.ac.uk

Abstract

A 3D model has been developed that combines bicycle mechanics, rider biomechanics and environmental factors into a single dynamic system. The aim of the model is to identify mechanical mechanisms that influence performance in a road cycling time trial with simulations representing actual cyclists competing in actual events.

The model is constructed using the Matlab toolbox SimMechanics to model physical entities and Simulink to model control structures. In SimMechanics, a 'machine' is built using blocks to represent rigid bodies linked by joints (including closed loops). The system is actuated by force or motion actuators applied to joints or bodies while sensors measure the resulting forces and motion. A range of constraint blocks allow limits to be placed on forces/motions and provide functions such as gears and rolling wheels. Rigid bodies and joints are linked with lines that essentially represent 2-way 'action-reaction' physical connections providing implicit inertial effects in a complete system. SimMechanics automatically derives the equations of motion leaving the developer free to concentrate on defining the mechanics of the system. Initial conditions are specified and a variable step ODE solver numerically integrates solutions that meet defined tolerances. The developed system operates in forward dynamics mode where forces applied to the model result in motion subject to constraints.

The main sub-systems comprising the model are shown in Table 1.

Table 1. Model sub-systems

Bicycle (Trek Madonne)	Rider (Typical 70 kg)	Environment (Course G10/42)
16 rigid bodies with dimension/mass/inertia	14 body segments (from literature)	Course track (from digital map)
Freedoms: x, y translation; roll, pitch, yaw rotation; steering, cranks/wheels rotation	Symmetrical two legged pedalling	Course gradient (from aerial laser mapping)
Holonomic + non-holonomic wheel constraints	Cyclic vertical/horizontal pedal force (phased 180°)	Bicycle/rider aerodynamics
Tyres (slip/camber forces, aligning/overturning moments)	Synchronised bicycle-rider roll	Environmental wind speed/direction
Geometry (COM, steer axis, trail, wheelbase)	Balance, counter-steering and path following	
Transmission		
Frame + wheel flex		

Initial model validation has been against the literature. Rider-less self-stability after a perturbation was simulated resulting in weave eigenvalues becoming negative at 4.2 m/s and capsize eigenvalues becoming slightly positive at 6.1 m/s. Both values compare well with the 2007 findings of Meijaard et al. [1] and the associated 2008 experimental validation by Kooijman et al [2]. Secondly, crank torque profile over 360° when the rider pedalled at 255 W was recorded from the model and found to correlate well (R^2=0.97) with experimental data reported in 1986 by Redfield and Hull [3]. Thirdly, the tyre model generated tyre cornering stiffness of 62N/degree (3 degrees slip, 4 degrees camber and 338 N vertical load) which closely matched the 60 N/degree (3 degrees slip, 10 degrees camber and 330 N vertical load) reported in 1972 by Roland and Lynch [4].

A field study analysed 20 experienced cyclists completing a time trial over an undulating 2.5 mile road course. The course was digitised and loaded into the model and the model parameterised with individual mass and aerodynamic characteristics. Wind strength and direction were recorded with an anemometer every 10 min during a trial and entered into the model. An error level of 1.4% (±1.5%) between actual and predicted individual time for the cyclist's 'best effort' over the course was obtained which is well below the 3.9% error for the road cycling model presented in 1995 by Olds et al. [6].

A subsequent investigation modelled and experimentally confirmed the theoretical advantage of adopting a variable rather than constant power strategy over an undulating road time trial course. The model generated an optimum power profile that minimised completion time subject to constraints on mean and peak power. Twenty time-trialists completed a total of 103 trials over the course utilising both strategies with the variable power requirement at ~80 m intervals dictated through an earpiece driven by a PDA/GPS attached to the arm. The model predicted a 4% time saving for the variable power strategy whereas a mean 2.9% actual saving was obtained in the field trials. The difference was partly attributable to unmeasured traffic drafting effects.

Further studies with the validated model are to investigate the mechanical performance advantages of bike/rider weight, saddle position, crank length, tyre characteristics and the contribution of muscular/non-muscular forces to pedalling.

The model is currently an incomplete representation of road cycling which is being addressed by the following enhancements:

- Asymmetrical pedalling
- Steering by rider arms
- Gears
- Vertical translation
- Magic Formula tyre model
- Pedalling by leg joint torques

References

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